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Modeling and solving the Tactical Berth Allocation Problem

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ABSTRACT

In this paper we integrate at the tactical level two decision problems arising in container terminals: the berth allocation problem, which consists of assigning and scheduling incoming ships to berthing positions, and the quay crane assignment problem, which assigns to incoming ships a certain quay crane profile (i.e. number of quay cranes per working shift). We present two formulations: a mixed integer quadratic program and a linearization which reduces to a mixed integer linear program. The objective function aims, on the one hand, to maximize the total value of chosen quay crane profiles and, on the other hand, to minimize the housekeeping costs generated by transshipment flows between ships. To solve the problem we developed a heuristic algorithm which combines tabu search methods and mathematical programming techniques. Computational results on instances based on real data are presented and compared to those obtained through a commercial solver.

1. Introduction

Maritime transportation has always played a crucial role in the exchange of goods between continents, and reducing the cost of such shipping continues to be an important commercial goal. Containerization has allowed for major improvements in the maritime transport cost structure (UNCTAD, 2008). In order to further reduce transportation costs, shippers seek to increase economies of scale, building ever larger container ships for long-haul routes, and demanding terminals with facilities and technologies able to handle them (mega-terminals). This system is known as *hub and spoke*: deep sea containerships (*mother vessels*) operate among a limited number of transhipment terminals (*hubs*), and smaller vessels (*feeders*) link the hubs with the other ports (*spokes*). The need for an efficient management of logistic activities at modern container terminals, and especially at the major hubs, is well recognized. For an overview and classification of the various equipments and decision problems in such systems, see Vis and de Koster (2003), Steenken et al. (2004), Crainic and Kim (2007), Stahlbock and Voss (2008) and Monaco et al. (2009).

Among the several related problems addressed in the literature, one of the most relevant is the well known Operational Berth Allocation Problem (OBAP), which consists of assigning and scheduling ships to berthing positions along the quay, with the aim of minimizing ships' turnaround time. The OBAP typically covers a planning horizon of at most 1 week, due to the uncertainties of maritime traveling times. This paper deals with a new model for the integration, at the tactical level, of the berth allocation problem with quadratic yard costs and the quay crane assignment problem. Our specific motivation in building a Tactical Berth Allocation Problem (TBAP), is not simply the obvious one of considering a longer planning horizon, but mainly that of supporting decisions made by terminal managers in the negotiation process with shipping lines. During this process, terminal managers need to evaluate the impact on the performance of the terminal of assigning certain operating

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resources, i.e., berths and quay cranes, to the shipping lines. Drawing inspiration from the actual negotiation process, the key feature of our model is the inclusion of a quay crane (QC) profile. This profile, which represents the number of quay cranes available to a berthed vessel at each time step, is explicitly modeled as a decision variable. While this will be clarified later in the paper, for the remainder of this section we highlight the main features of TBAP assuming the reader to be familiar with existing OBAP formulations.

Basically, both the tactical and the operational problems deal with assigning and scheduling ships to berthing positions, i.e. deciding where and when the ships should moor. Both the TBAP and the OBAP aim to balance terminal costs and service quality. However, as already noted, the different decision levels and time frames induce different problems. In the TBAP service quality depends upon the negotiation between the terminal and the shipping lines regarding the terminal resources. A higher service quality occurs when the terminal can accommodate shipping lines requests in terms of expected berthing times, assigned quay cranes, and expected handling times. In the TBAP, for a given amount of requested QC hours, it could be possible to create different QC profiles. For example, assume that we have a request for a vessel that requires six QCs work shifts, and the customer is potentially willing to accept either an intensive profile (for example three QCs on two work shifts) or a longer one (two QCs on three work shifts). The terminal managers want to know the trade-off between the two profiles. The faster one will be likely more satisfying for the customer because of the smaller handling time; while the slower one will put less pressure on the quay cranes availability, which could be a bottleneck in some periods. However, the problem is more complicated because if the QC availability is not a limiting factor, then a faster handling time is advantageous for the terminal, because it augments berth availability. This shows why the Quay Crane Assignment Problem (QCAP), i.e. deciding how many QCs to assign and for how long, has an impact on the berth allocation.

The TBAP, thanks to the longer planning horizon, can optimize the terminal's total costs in a more comprehensive way. In a transshipment terminal, containers arrive and depart on vessels while being temporarily stored in the yard. When unloading a vessel, the discharged containers must be allocated to yard positions close enough to the vessel berthing point in order to speed up the vessel handling. However, when the departure position of a container is far from its yard position, the container must be reallocated before the arrival of the outbound vessel. In the OBAP, since the planning horizon is shorter than the average container dwell time inside the yard, one can assume that the majority of the outbound containers are already in the yard, and disregard the effects of transshipment flows inside the yard. In the TBAP, the yard costs cannot be simplified by this assumption, and a quadratic term must be considered to account for the simultaneous assignment of vessels to berths.

In the following we present the paper outline, and we highlight its contributions. A literature review is provided in Section 2 while the problem description as well as two formulations for the TBAP are presented in Section 3. With respect to mathematical modeling, the paper contributions are the introduction of the quay crane assignment profile concept, detailed in Section 3.1, and the integration with yard issues in transshipment terminals, discussed in Section 3.2. Both quay crane assignment profiles, and yard aspects, have motivated the new optimization model presented in Section 3.3. The advantages of the new model consists in the simultaneous control of the terminal on critical resources such as berths and quay cranes. From a computational point of view, the paper contribution relies upon the solution algorithm introduced in Section 4 which is based on mathematical programming techniques employed in a two-level heuristic. While the mathematical model is not addressable by a state-of-the-art solver, the proposed heuristic proves its efficacy as documented by the computational experiments reported in Section 5. Finally, we draw our conclusions in Section 6.

2. Literature review

The OBAP consists of allocating ships to berths along a time axis. Usual side constraints are berth's allowable draft (depth of the water), time windows and priorities assigned to the ships, favorite berthing areas, etc. The OBAP can be modeled as a discrete problem if the quay is viewed as a finite set of berths. In this case the berths can be described as fixed length segments, or, if the spatial dimension is ignored, as points. Continuous models consider that ships can berth anywhere along the quay. While continuous models are more realistic, discrete ones can be very useful to study relaxed problems in order to devise efficient algorithms for them. The operational berth allocation problem has received so far a larger attention than the tactical one in the scientific literature, see e.g. Imai et al. (1997, 2001, 2003, 2005, 2007), Lim (1998), Nishimura et al. (2001), Kim and Moon (2003), Cordeau et al. (2005), Monaco and Sammarra (2007) and Wang and Lim (2007). In the following we discuss in more detail the articles relevant to the TBAP.

Moorthy and Teo (2006) address the design of a berth template, a tactical planning problem that arises in transshipment hubs and concerns the allocation of favorite berthing locations (home berths) to vessels which periodically call at the terminal. The problem is modeled as a bicriteria optimization problem, which reflects the trade-off between service levels and costs. The authors propose two procedures able to build good and robust templates, which are evaluated by simulating their performances; robust templates are also compared with optimal templates on real-life generated instances. The paper approach builds on a heuristic algorithm for the OBAP presented in Dai et al. (2007).

Cordeau et al. (2007) provide an initial introduction for the basis of the TBAP. The paper deals with the Service Allocation Problem (SAP), a tactical problem arising in the yard management of a container transhipment terminal. A *service*, also called *port route*, is the sequence of ports visited by a vessel. Shipping companies plan their port routes in order to match the demand for freight transportation. A shipping company will usually ask the terminal management to dedicate specific areas of the yard and the quay (home berths) to their services. The SAP objective is the minimization of container rehandling operations inside the yard through choosing the home berth for each service. The SAP is formulated as a Generalized Quadratic Assignment Problem (GQAP, see e.g. Cordeau et al. (2006) and Hahn et al. (2008)) with side constraints, and solved by an evolutionary heuristic. The SAP can be seen as a relaxed TBAP when collapsing the temporal dimension, and disregarding the choice of QC profiles. The SAP output consists of reference home berths that planners consider when drawing the berth template.

Park and Kim (2003) were the first to integrate the OBAP in the continuous case with the QCAP, also considering the scheduling of quay cranes. The integrated problem is formulated as an integer program and a two-phase solution procedure is presented to solve the model. In the first phase, the berthing time and position of vessels and the number of quay cranes assigned to each vessel at each time step are determined using Lagrangian relaxation and a subgradient optimization technique; the objective is to minimize the sum of penalty costs over all ships. In the second phase, cranes are scheduled along the quay via dynamic programming, with the objective of minimizing the number of setups. With respect to the problem formulation, the authors take into account some practical aspects such as favorite berthing positions of vessels, maximum and minimum number of cranes to be assigned to each vessel, penalty costs due to earlier or later berthing time, and later departure time (with respect to previously committed time).

Meisel and Bierwirth (2006) investigate the simultaneous allocation of berths and quay cranes, focusing on the reduction of QCs idle times, which significantly impact on terminal's labor costs. A heuristic scheduling algorithm based on priority-rules methods for resource-constrained project scheduling is proposed and tested on instances based on real data. Preliminary results, compared to the manually generated schedules which have been used in practice, are encouraging. In this approach, each vessel represents an activity which can be performed in eight different modes, each mode representing a given QC-to-Vessel assignment over time. The concept of "mode" seems analogous to the concept of profile we have introduced so far; however, no detailed description of these modes is available in the paper.

Imai et al. (2008) address the simultaneous berth–crane allocation and scheduling problem, taking into account physical constraints of quay cranes, which cannot move freely among berths as they are all mounted on the same track and cannot bypass each other. A MIP formulation which minimizes the total service time is proposed and a genetic algorithm-based heuristic is developed to find an approximate solution. As authors recognize, the relationship between the number of cranes and the handling time is not investigated in the paper; indeed, a reference number of cranes needed by each ship is assumed to be given as input of the problem.

Meisel and Bierwirth (2009) study the integration of OBAP and QCAP with a focus on quay crane productivity. An integer linear model is presented and a construction heuristic, local refinement procedures and two meta-heuristics are developed to solve the problem. Authors compare their approach to the one proposed by Park and Kim (2003) over the same set of instances and they always provide better solutions. An analysis of quay crane's productivity losses, mainly due to interference among QCs and to the distance of the vessel berthing position from the yard areas assigned to this vessel, is also presented and their impact on the terminal's service cost is evaluated.

3. Mathematical models

In this section we provide a compact description of the problem and motivate our modeling choices. In particular, in Section 3.1, we illustrate the concept of QC assignment profiles, and in Section 3.2 we provide additional details regarding yard costs related to transshipment flows among ships. The described cost figures and operational parameters were provided by the Medcenter Container Terminal (MCT), port of Gioia Tauro, Italy. We then present a mixed integer quadratic programming formulation (MIQP) for the TBAP with integrated QCs assignment in Section 3.3, as well as a linearization of the MIQP model which reduces to a mixed integer linear program (MILP) in Section 3.4.

With respect to the OBAP, we consider the discrete case. As described in Section 1, the fundamental modeling tool of our formulation is the quay crane profile, representing the number of quay cranes assigned to the ship at each time step. Given n ships, m berths, and a particular time horizon, we aim to assign a home berth and a QC profile to each ship, as well as schedule incoming ships according to time windows on their arrival time and on berths' availabilities. These decisions are made to maximize the total value of chosen QC assignment profiles and minimize the housekeeping costs generated by transshipment flows between ships.

The integrated problem presents increased complexity because the ship handling time is not constant but depends on the number of quay cranes assigned to the ship. With respect to the classical OBAP, this implies additional decision variables and constraints.

3.1. QC assignment profiles

The use of QC profiles to handle the assignment of quay cranes to ships is firstly motivated by the needs of terminal managers during negotiations with shipping companies. In particular, managers need to be aware of the trade-off among the different QC profiles they may propose to the shippers.

Concerning the mathematical model, the concept of QC profiles can capture real-world issues, and works well to represent the control that the terminal has on several aspects of QC assignment during the optimization process. These are the main reasons why we have explicitly introduced this feature in the formulation.

We assume to have a set of feasible QC profiles P_i for every ship $i \in N = 1, \ldots, n$, which are defined by the terminal according to the specific amount of QC hours requested by the ship, internal rules and good practices related to the efficiency of terminal operations, and legal contracts.

Our approach differs from the traditional modeling choice present in the literature, e.g. Park and Kim (2003), Imai et al. (2008) and Meisel and Bierwirth (2009), which usually assigns quay cranes hour by hour, without any control on the final outcome in terms of QC profiles. As mentioned, the concept of "mode" in Meisel and Bierwirth (2006) is somehow similar to our concept of QC profile, but the authors do not provide enough details to allow comparisons.

For a given vessel, feasible QC profiles usually vary in length (number of shifts) as well as in the distribution of QC cranes over the active shifts, in order to ensure the requested amount of QC hours.

Some operational constraints, which are usually not taken into account by other models, can be directly integrated in the definition of the set of feasible profiles. A common rule, for instance, is that quay cranes are assigned to vessels and placed on the corresponding quay segment shift by shift. This means that a quay crane cannot be moved from one vessel to another at any arbitrary moment, but only between two shifts. This constraint can be easily handled by forcing profiles to maintain a constant number of quay cranes during a shift. Another good practice is to keep the distribution of quay cranes as regular as possible among active shifts; a variance of one or at most two QCs can be considered acceptable, although high variability should be avoided as much as possible. Also this feature can be included in our profile set definition easily.

In addition to these general rules, the terminal can manage more directly some priority-related issues. Since the set of feasible QC profiles is defined for every ship, managers can assign different minimum and maximum handling times not only depending on the ship's size and the traffic volume but also depending on the ship's relative importance for the terminal. This also applies for the minimum and maximum number of quay cranes allowed to be assigned to a given ship. We would like to remark that this is an important advantage provided by our approach, compared to other models in the literature where handling time is either considered an input of the problem or barely controlled by time windows on the vessel's arrival and departure, in addition to some priority-related weights in the objective function, which usually aim to serve faster vessels with high priority. Furthermore, each QC profile has an associated "value" which reflects technical aspects (such as the resources utilized) but which is also computed by taking into account the specific vessel which will use the profile. In other words, the same QC profile can have different values when applied to different ships, according to their priority or importance.

We can also include productivity losses due to quay crane interference, recently studied by Meisel and Bierwirth (2009), in the definition of the feasible set of profiles. Indeed, we can use the approach suggested by the authors to compute, for each profile, the actual quay crane productivity instead of the theoretical one.

In order to improve understanding of the QC profile concept, and its relation with the integration between Berth and QC Allocation planning, we provide in Fig. 1 an example of such a plan. The example is for the scheduling and assignment of five vessels to three berths over a time horizon of eight working shifts. Consider, for instance, the Ship 1: it berths at shift 1, and three QCs are allocated to it for carrying out operations during the same shift; next, at working shifts 2 and 3, Ship 1 remains berthed, but one QC is de-allocated, with only two QCs remaining allocated to the ship. At the end of shift 3 operations terminate and the ship is released.

3.2. Transshipment-related yard costs

When loading (or unloading) a vessel, the containers must be at (or allocated to) yard positions close enough to the vessel berthing point to maintain the required speed of quay crane operation. Usually, at the Medcenter Container Terminal (MCT) of the port of Gioia Tauro, a yard position is evaluated as satisfyingly close to a berth if the distance along the quay axis is less than 600 m. This maximal close distance value can be lowered for higher priority workloads. Furthermore, when we estimate yard-related transshipment costs induced by berth allocation, we do not consider the real yard position of the loading and unloading containers. In fact, we assume that the expected travelled distance along the quay axis is given by the distance

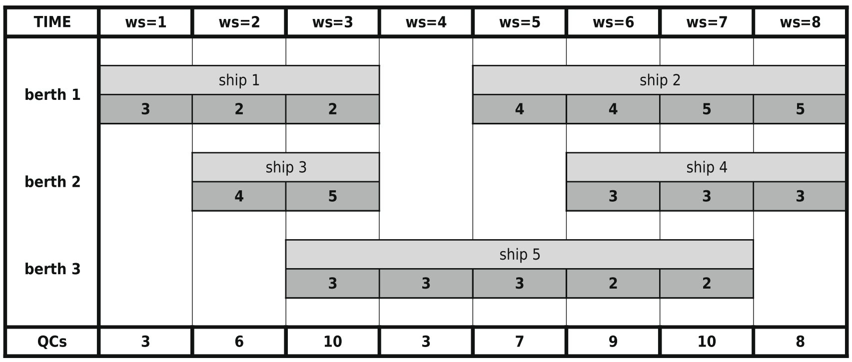

Fig. 1. Example of a berth and quay cranes allocation plan.

between the incoming and outgoing berths. If this distance is lower than the threshold value of 600 m, then a container will likely move from the quay to its assigned yard position when unloading and from this yard position to the quay when loading. However, in a large transshipment terminal, such as the one at the Gioia Tauro port, the distance between the unloading berth and the loading one is often larger than 600 m. Therefore, containers are moved before the arrival of the outgoing vessel from their current yard positions to new ones closer to the outgoing berth. This process is called *housekeeping* and requires a dedicated management in order to accommodate operational constraints like the capacity of the yard positions, the maximum container handling workload for a given work shift, etc. In synthesis, the yard management deals with a dynamic allocation of containers through their duration-of-stay inside the terminal, see Moccia et al. (2009). A rule motivated by cost minimization enforces that whenever the distance along the quay axis is larger than 1100 m, the yard-to-yard transfer is operated by deploying multi trailer vehicles instead of straddle carriers. Therefore, we have a yard cost function that depends upon the distance between the incoming and outgoing berths according to three transport modalities:

- the distance is below 600 m: no housekeeping is performed, the unitary transport cost, euro/(meter $\times$ container), depends upon straddle carriers cost figures only;
- the distance is between 600 and 1100 m: a housekeeping process is activated by deploying straddle carriers only, however, we face a transport cost larger than in the previous distance range;
- the distance is larger than 1100 m: the housekeeping is performed by using the less expensive multi trailer vehicles (higher capacity than the straddle carriers).

The qualitative pattern of this piecewise linear cost function is given in Fig. 2, where we indicate by SC the direct transfer with straddle carriers, by HK SC the housekeeping with straddle carriers, and by HK MT the housekeeping with multi trailer vehicles.

3.3. MIQP formulation

In this section we present a mixed integer quadratic programming formulation for the TBAP with QCs assignment. Input data for this problem are:

N	set of vessels, with $\lvert N\rvert = n$
M	set of berths, with $\lvert M\rvert = m$
H	set of time steps (each time step $h \in H$ is submultiple of the work shift length)
S	set of the time step indexes $\{1,\ldots,\bar{s}\}$ relative to a work shift; $\bar{s}$ represents the number of time steps in a work shift
H^s	subset of H which contains all the time steps corresponding to the same time step $s \in S$ within a work shift
P_i^s	set of feasible quay crane assignment profiles for the vessel $i \in N$ when vessel arrives at a time step with index $s \in S$ within a work shift
P_i	set of quay crane assignment profiles for the vessel $i \in N$, where $P_i = \cup_{s\in S} P_i^s$
t_i^p	handling time of ship $i \in N$ under the QC profile $p \in P_i$ expressed as multiple of the time step length
v_i^p	the value of serving the ship $i \in N$ by the quay crane profile $p \in P_i$
q_i^{pu}	number of quay cranes assigned to the vessel $i \in N$ under the profile $p \in P_i$ at the time step $u \in (1,\ldots,t_i^p)$, where $u = 1$ corresponds to the ship arrival time
Q^h	maximum number of quay cranes available at the time step $h \in H$
f_{ij}	number of containers exchanged between vessels $i,j \in N$
d_{kw}	unit housekeeping cost between yard slots corresponding to berths $k,w \in M$
$[a_i, b_i]$	[earliest, latest] feasible arrival time of ship $i \in N$
$[a^k, b^k]$	[start, end] of availability time of berth $k \in M$
$[a^h, b^h]$	[start, end] of the time step $h \in H$

We define a graph $G^k = (V^k, A^k)\ \forall k \in M$, where $V^k = N \cup \{o(k), d(k)\}$, with $o(k)$ and $d(k)$ additional vertices representing berth k, and $A^k \subseteq V^k \times V^k$. The following decision variables are defined:

- $x_{ij}^k \in \{0,1\}\ \forall k \in M,\ \forall (i,j) \in A^k$, set to 1 if ship j is scheduled after ship i at berth k, and 0 otherwise;
- $y_i^k \in \{0,1\}\ \forall k \in M,\ \forall i \in N$, set to 1 if ship i is assigned to berth k, and 0 otherwise;
- $\gamma_i^h \in \{0,1\}\ \forall h \in H,\ \forall i \in N$, set to 1 if ship i arrives at time step h, and 0 otherwise;
- $\lambda_i^p \in \{0,1\}\ \forall p \in P_i,\ \forall i \in N$, set to 1 if ship i is served by the profile p, and 0 otherwise;
- $\rho_i^{ph} \in \{0,1\}\ \forall p \in P_i,\ \forall h \in H, \forall i \in N$, set to 1 if ship i is served by profile p and arrives at time step h, and 0 otherwise;
- $T_i^k \geqslant 0\ \forall k \in M,\ \forall i \in N$, representing the berthing time of ship i at the berth k, i.e. the time when the ship moors;
- $T_{o(k)}^k \geqslant 0\ \forall k \in M$, representing the starting operation time of berth k, i.e. the time when the first ship moors at the berth;
- $T_{d(k)}^k \geqslant 0\ \forall k \in M$, representing the ending operation time of berth k, i.e. the time when the last ship departs from the berth.

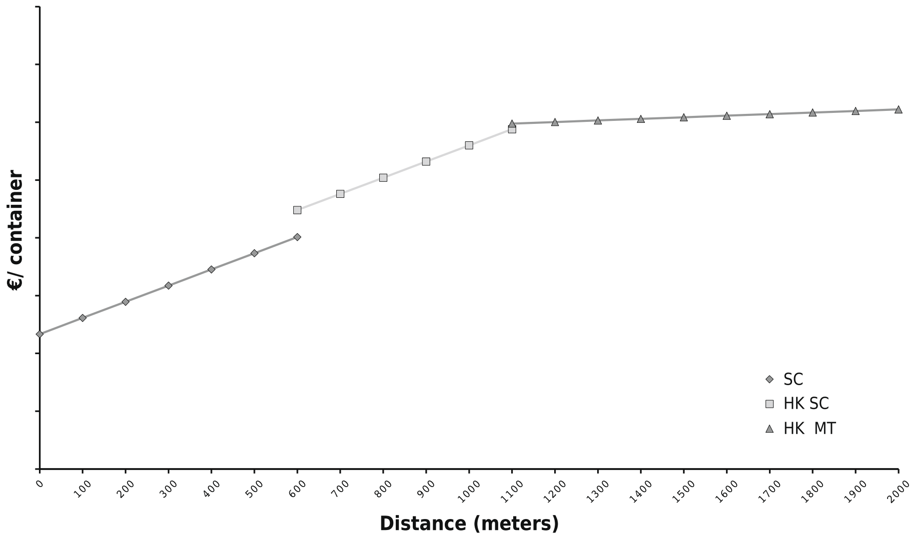

Fig. 2. Yard costs according to the distance between the incoming and outgoing berths.

The TBAP with QC assignment can, therefore, be formulated as follows:

$$\max \quad \sum_{i\in N}\sum_{p\in P_i} v_i^p \lambda_i^p - \frac{1}{2}\sum_{i\in N}\sum_{k\in M} y_i^k \sum_{j\in N}\sum_{w\in M} f_{ij} d_{kw} y_j^w \tag{1}$$

$$\text{s.t.} \quad \sum_{k\in M} y_i^k = 1 \quad \forall i\in N, \tag{2}$$

$$\sum_{j\in N\cup\{d(k)\}} x_{o(k),j}^k = 1 \quad \forall k\in M, \tag{3}$$

$$\sum_{i\in N\cup\{o(k)\}} x_{i,d(k)}^k = 1 \quad \forall k\in M, \tag{4}$$

$$\sum_{j\in N\cup\{d(k)\}} x_{ij}^k - \sum_{j\in N\cup\{o(k)\}} x_{ji}^k = 0 \quad \forall k\in M,\ \forall i\in N, \tag{5}$$

$$\sum_{j\in N\cup\{d(k)\}} x_{ij}^k = y_i^k \quad \forall k\in M,\ \forall i\in N, \tag{6}$$

$$T_i^k + \sum_{p\in P_i} t_i^p \lambda_i^p - T_j^k \leqslant \left(1 - x_{ij}^k\right) M1 \quad \forall k\in M,\ \forall i\in N,\ \forall j\in N\cup\{d(k)\}, \tag{7}$$

$$T_{o(k)}^k - T_j^k \leqslant \left(1 - x_{o(k),j}^k\right) M2 \quad \forall k\in M,\ \forall j\in N, \tag{8}$$

$$a_i y_i^k \leqslant T_i^k \quad \forall k\in M,\ \forall i\in N, \tag{9}$$

$$T_i^k \leqslant b_i y_i^k \quad \forall k\in M,\ \forall i\in N, \tag{10}$$

$$a^k \leqslant T_{o(k)}^k \quad \forall k\in M, \tag{11}$$

$$T_{d(k)}^k \leqslant b^k \quad \forall k\in M, \tag{12}$$

$$\sum_{p\in P_i} \lambda_i^p = 1 \quad \forall i\in N, \tag{13}$$

$$\sum_{h\in H^s} \gamma_i^h = \sum_{p\in P_i^s} \lambda_i^p \quad \forall i\in N,\ \forall s\in S, \tag{14}$$

$$\sum_{k\in M} T_i^k - b^h \leqslant (1-\gamma_i^h) M3 \quad \forall h\in H,\ \forall i\in N, \tag{15}$$

$$a^h - \sum_{k\in M} T_i^k \leqslant (1-\gamma_i^h) M4 \quad \forall h\in H,\ \forall i\in N, \tag{16}$$

$$\rho_i^{ph} \geqslant \lambda_i^p + \gamma_i^h - 1 \quad \forall h\in H,\ \forall i\in N,\ \forall p\in P_i, \tag{17}$$

$$\sum_{i\in N}\sum_{p\in P_i}\sum_{u=max\{h-t_i^p+1;1\}}^{h} \rho_i^{pu} q_i^{p(h-u+1)} \leqslant Q^h \quad \forall h\in H^{\bar{s}}, \tag{18}$$

$$x_{ij}^k \in \{0,1\} \quad \forall k \in M,\ \forall (i,j) \in A^k, \tag{19}$$
$$y_i^k \in \{0,1\} \quad \forall k \in M,\ \forall i \in N, \tag{20}$$
$$\gamma_i^h \in \{0,1\} \quad \forall h \in H,\ \forall i \in N, \tag{21}$$
$$\lambda_i^p \in \{0,1\} \quad \forall p \in P_i,\ \forall i \in N, \tag{22}$$
$$\rho_i^{ph} \in \{0,1\} \quad \forall p \in P_i,\ \forall h \in H,\ \forall i \in N, \tag{23}$$
$$T_i^k \geqslant 0 \quad \forall k \in M,\ \forall i \in N \cup \{o(k), d(k)\}. \tag{24}$$

where M1, M2, M3 and M4 are sufficiently large constants. The objective function (1) maximizes the sum of the values of the chosen quay crane assignment profiles over all the vessels and simultaneously minimizes the yard-related housekeeping costs generated by the flows of containers exchanged between vessels. Constraints (2) state that every ship i must be assigned to one and only one berth k. Constraints (3) and (4) define the outgoing and incoming flows to the berths, while flow conservation for the remaining vertices is ensured by constraints (5). Constraints (6) state the link between variables x_{ij}^k and y_i^k, while precedences in every sequence are ensured by constraints (7) and (8), which coherently set time variables T_i^k. Time windows on the arrival time are stated for every ship by constraints (9) and (10), while time windows on berths' availabilities are stated by constraints (11) and (12). Constraints (13) ensure that one and only one QC profile is assigned to every ship. Constraints (14) define the link between variables γ_i^h and λ_i^p while constraints (15) and (16) link binary variables γ_i^h to the arrival time T_i^k. Observe that constraints (10) imply $T_i^k = 0$ when ship $i \in N$ does not moor at berth $k \in K$. Variables ρ_i^{ph} are linked to variables λ_i^p and γ_i^h by constraints (17): in particular, ρ_i^{ph} is equal to 1 if and only if $\lambda_i^p = \gamma_i^h = 1$. Finally, constraints (18) ensure that, at every time step, the total number of assigned quay cranes does not exceed the number of quay cranes which are available in the terminal.

To better illustrate capacity constraints (18), we come back to the example shown in Fig. 1, which refers to the scheduling and assignment of $|N| = 5$ vessels to $|M| = 3$ berths over a time horizon of $|H| = 8$ time steps. Here, we assume that a time step corresponds to one working shift. From the plan we can infer the following non-zero data:

$$\begin{array}{llllllll}
i=1 & \rho_1^{p1}=1 & t_1^p=3 & q_1^{p1}=3, & q_1^{p2}=2, & q_1^{p3}=2 & & \\
i=2 & \rho_2^{p5}=1 & t_2^p=4 & q_2^{p1}=4, & q_2^{p2}=4, & q_2^{p3}=5, & q_2^{p4}=5 & \\
i=3 & \rho_3^{p2}=1 & t_3^p=2 & q_3^{p1}=4, & q_3^{p2}=5 & & & \\
i=4 & \rho_4^{p6}=1 & t_4^p=3 & q_4^{p1}=3, & q_4^{p2}=3, & q_4^{p3}=3 & & \\
i=5 & \rho_5^{p3}=1 & t_5^p=5 & q_5^{p1}=3, & q_5^{p2}=3, & q_5^{p3}=3, & q_5^{p4}=2, & q_5^{p5}=2
\end{array}$$

For each time step $h = 1, \ldots 8$, the corresponding constraint in (18) counts the number of active quay cranes. Let us consider the case $h = 3$: the index u changes its range for each vessel, because, starting from $h = 3$, it goes backwards until the beginning of the profile. Therefore, we have:

$$\begin{array}{ll}
i=1 & u=1,2,3 \\
i=2 & u=1,2,3 \\
i=3 & u=2,3 \\
i=4 & u=1,2,3 \\
i=5 & u=1,2,3
\end{array}$$

We remark that vessels $i = 2,\ 4$ do not contribute to the sum, since $\rho_2^{pu} = \rho_4^{pu} = 0\ \forall u = 1,2,3$ and this is coherent with the plan. For the remaining vessels, ρ_i^{pu} is not zero only for one value u^*:

$$\begin{array}{ll}
i=1 & u^*=1 \Rightarrow q_1^{p(3-1+1)} = q_1^{p3} = 2 \\
i=3 & u^*=2 \Rightarrow q_3^{p(3-2+1)} = q_3^{p2} = 5 \\
i=5 & u^*=3 \Rightarrow q_5^{p(3-3+1)} = q_5^{p1} = 3
\end{array}$$

Therefore, the sum in (18) reduces to:

$$q_1^{p3} + q_3^{p2} + q_5^{p1} = 2 + 5 + 3 = 10$$

which is indeed the total number of quay cranes which are active at time step $h = 3$.

Finally, we observe that the TBAP formulation (1)–(24) can be interpreted as a Multi-Depot Vehicle Routing Problem with Time Windows (MDVRPTW), see e.g. Cordeau et al. (2005), with an additional quadratic component in the objective function and side constraints.

3.4. MILP formulation

The quadratic objective function (1) can be linearized by defining an additional decision variable $z_{ij}^{kw} \in \{0,1\}\ \forall i,j \in N,\ \forall k,w \in M$, which is equal to 1 if $y_i^k = y_j^w = 1$ and 0 otherwise. Variables z_{ij}^{kw} are linked to variables y_i^k by the following additional constraints:

$$\sum_{k\in K}\sum_{w\in K} z_{ij}^{kw} = g_{ij} \quad \forall i,j \in N, \tag{25}$$

$$z_{ij}^{kw} \leqslant y_i^k \quad \forall i,j \in N,\ \forall k,w \in M, \tag{26}$$

$$z_{ij}^{kw} \leqslant y_j^w \quad \forall i,j \in N,\ \forall k,w \in M, \tag{27}$$

where g_{ij} is a constant which is equal to 1 if $f_{ij} > 0$ and 0 otherwise.

TBAP can, therefore, be formulated as a mixed integer linear program as follows:

$$\max \quad \sum_{i\in N}\sum_{p\in P_i} v_i^p \lambda_i^p - \frac{1}{2}\sum_{i\in N}\sum_{j\in N}\sum_{k\in M}\sum_{w\in M} f_{ij} d_{kw} z_{ij}^{kw} \tag{28}$$

$$\text{s.t.} \quad (2){-}(27).$$

4. A two-level heuristic for TBAP

Solving the TBAP model with a general-purpose solver is difficult, using either the MIQP or MILP formulation, as shown by computational results in Section 5. A specialized heuristic is, therefore, needed. We propose a two-level heuristic algorithm for solving the TBAP, which is illustrated in this section.

Our heuristic is organized in two stages: first, we identify a QC profiles' assignment for the ships; second, we solve the resulting berth allocation problem for the given QC assignment. This procedure is repeated for several QC profiles, which are chosen, iteration by iteration, using the traditional reduced costs arguments of mathematical programming. A scheme of the heuristic algorithm for TBAP is outlined in Fig. 3.

The initialization consists of assigning a QC profile to each ship. The maximum value profile is chosen for each ship (ties are broken arbitrarily). This is equivalent to assign binary values to variables λ such that Eq. (13) are satisfied. Once the first QC profiles' assignment has been done, the two-level procedure starts.

Given a QC assignment, the TBAP reduces to the berth allocation problem, with additional constraints due to the QC total capacity. We developed a tabu search algorithm which solves the berth allocation problem (BAP) (step 1 in Fig. 3), aiming to minimize the yard-related transshipment housekeeping costs:

$$\frac{1}{2}\sum_{i\in N}\sum_{k\in M} y_i^k \sum_{j\in N}\sum_{w\in M} f_{ij} d_{kw} y_j^w. \tag{29}$$

We remark that we take into account only the quadratic term of the TBAP objective function in (1) since, for a given QC profiles' assignment, the total value of profiles is constant. The tabu search algorithm for the BAP is illustrated in Section 4.1.

In step 2, the QC profiles' assignment vector is updated. The new set of profiles is determined using the reduced costs of variables λ, whose estimation is illustrated in Section 4.2.

4.1. Tabu search for the berth allocation

Our tabu search heuristic is an adaptation of the one of Cordeau et al. (2005) for the OBAP. However, while in Cordeau et al. (2005) the function to be minimized is the weighted sum for every ship of the service time in the port, our heuristic minimizes the yard-related housekeeping costs generated by the flows of containers exchanged between vessels. Another difference is the handling of the side constraints concerning the QC availability for a given assignment of QC profiles (vector λ), our tabu search must take into account the QC capacity constraints (18).

Denote by S the set of solutions that satisfy constraints (2)–(9) and (11). The heuristic explores the solution space by moving at each iteration from the current solution s to the best solution in its neighborhood $N(s)$. Each solution $s \in S$ is represented by a set of m berth sequences such that every ship belongs to exactly one sequence. This solution may, however, violate the time window constraints associated with the ships and the berths, and the QC availability. The time window constraint on ship i on a berth k is violated if the arrival time T_i^k of the ship is larger than the time window's upper bound b_i.

Algorithm 1: TBAP Two-level Heuristic

Initialization : Assign a QC profile to each ship.
repeat

1. solve BAP via tabu search;
2. update the QC profiles' assignment by using reduced costs.

until *stopping criterion* ;

Fig. 3. Scheme of the heuristic algorithm for TBAP.

Berthing before a_i is not allowed; in other words, $T_i^k \geqslant a_i$. Similarly, the time window of berth k is violated when the completion time of a ship i assigned to berth k is larger than the berth time window's upper bound b^k.

Let $c(s)$ denote the cost of solution defined in (29), and let $w_1(s)$ denote the total violation of ships' time window constraints, equal to the sum of the violations on the n ships. We indicate as $w_2(s)$ the total violation of berths' time window constraints, equal to the sum of the violations on the m berths. Finally, let $w_3(s)$ be the total violation of QC availability for each time step of the planning horizon. Solutions are then evaluated by means of a penalized cost function $f(s) = c(s) + \alpha_1 w_1(s) + \alpha_2 w_2(s) + \alpha_3 w_3(s)$, where the α values are positive parameters. By dynamically adjusting the value of these parameters, the relaxation mechanism facilitates the exploration of the search space and is particularly useful for tightly constrained instances.

The tabu search method is based on the definition of attributes used to characterize the solutions of S. They are also used to control tabu tenures and to implement a diversification strategy. An attribute set $B(s) = \{(i,k)$: ship i is assigned to berth $k\}$ is associated to each solution $s \in S$. The neighborhood $N(s)$ of a solution s is defined by applying a simple operator that removes an attribute (i,k) from $B(s)$ and replaces it with another attribute (i,k'), where $k \neq k'$. When ship i is removed from berth k, the sequence is simply reconnected by linking the predecessor and successor of the ship. Insertion in sequence k' is then performed between two consecutive ships so as to minimize the value of $f(s)$. When a ship i is removed from berth k, its reinsertion in that berth is forbidden for the next θ iterations by assigning a tabu status to the attribute (i,k).

An aspiration criterion allows the revocation of the tabu status of an attribute if that would allow the search process to reach a solution of smaller cost than that of the best solution identified having that attribute. To diversify the search, any solution $\bar{s} \in N(s)$ such that $f(\bar{s}) \geqslant f(s)$ is penalized by a factor proportional to the addition frequency of its attributes, and by a scaling factor. More precisely, let ξ_{ik} be the number of times attribute (i,k) has been added to the solution during the process and let ζ be the number of the current iteration. A penalty $p(\bar{s}) = \beta c(\bar{s})\xi_{ik}/\zeta$ is added to $f(\bar{s})$. The scaling factor $c(\bar{s})$ introduces a correction to adjust the penalties with respect to the total solution cost. Finally, the parameter β is used to control the intensity of the diversification. These penalties have the effect of driving the search process toward less explored regions of the search space. For notational convenience, assume that $p(\bar{s}) = 0$ if $f(\bar{s}) < f(s)$.

In order to generate a starting solution, the algorithm assigns the ships to the berths at random. This initial solution is constructed by relaxing the time window and QC availability constraints, and, therefore, it is usually infeasible. However, this is not an issue for the tabu search heuristic.

The search starts from this initial solution and selects, at each iteration, the best non-tabu solution $\bar{s} \in N(s)$. After each iteration, the value of parameters α_1, α_2, and α_3 are modified by a factor $1 + \delta$, where $\delta > 0$. For example, if the current solution is feasible with respect to ships' time window constraints, the value of α_1 is divided by $1 + \delta$; otherwise, it is multiplied by $1 + \delta$. Analogously for the berths' time window and QC availability constraints, i.e. parameters α_2 and α_3, respectively. This process is repeated for η iterations and the best feasible solution s^* is updated throughout the search.

4.2. Profile update via mathematical programming

The profiles' updating procedure represents step 2 in the algorithm's scheme illustrated in Fig. 3. It relies on the MILP formulation for TBAP illustrated in Section 3.4. The basic idea of this step is to use the information of reduced costs in order to be able to update vector λ of QC profiles' assignment in a smart way.

Let $s^* = [\bar{x}, \bar{y}, \overline{T}]$ be the BAP solution provided by tabu search for a given QC profile assignment $\bar{\lambda}$. In particular, we are interested in reduced costs of variables λ, which we denote $\tilde{c}(\lambda)$. We remark that a BAP solution plus a QC assignment represent a feasible solution for TBAP. At each iteration, we solve the linear relaxation of the MILP formulation, with the additional constraints:

$$\bar{x} - \epsilon \leqslant x \leqslant \bar{x} + \epsilon, \tag{30}$$

$$\bar{y} - \epsilon \leqslant y \leqslant \bar{y} + \epsilon, \tag{31}$$

$$\overline{T} - \epsilon \leqslant T \leqslant \overline{T} + \epsilon, \tag{32}$$

$$\bar{\lambda} - \epsilon \leqslant \lambda \leqslant \bar{\lambda} + \epsilon. \tag{33}$$

As remarked, e.g., by Desrosiers and Lübbecke (2005), the shadow prices of constraints (30)–(33) are the reduced costs of original variables x, y, T and λ. At each iteration, we identify the $\lambda_{i^*}^{p^*}$ variable with the maximum reduced cost:

$$(i^*, p^*) = \arg \max_{i \in N, p \in P_i} \{\tilde{c}(\lambda_i^p)\}. \tag{34}$$

If $\tilde{c}\left(\lambda_{i^*}^{p^*}\right) > 0$, profile p^* is assigned to vessel i^*, i.e. $\lambda_{i^*}^{p^*} = 1$ and $\lambda_{i^*}^{p} = 0 \ \forall p \neq p^*$. We remark that this update, concerning a single vessel, results in a new vector λ of QC profiles' assignment, which differs from the previous one only for two components. Step 2 of the algorithm is now completed. The updated QC profiles' assignment vector is passed back to the BAP tabu search (step 1) and a new BAP solution is computed.

The whole procedure terminates when all reduced costs are non-positive, or other additional stopping criteria are reached, such as the maximum number of iterations or the time limit.

In order to prevent cycles, a tabu mechanism has been implemented, keeping track of the last ψ updates in the form (i,p). The tabu list (TL) is updated at each iteration and its length has been fixed to $\psi = 0.5n \times \bar{p}$, where $\bar{p} = \max_{i \in N}|P_i|$. According to this mechanism, the pair (i^*, p^*) is, therefore, chosen as

$$(i^*, p^*) = \arg \max_{i \in N, p \in P_i : (i,p) \notin TL} \{\tilde{c}(\lambda_i^p)\}.$$

It may happen that the tabu search returns a BAP solution which is infeasible for TBAP with respect to time windows and/or the QC availability. In this case the profiles' update via mathematical programming cannot be performed. We, therefore, update the set of profiles by randomly assigning a new QC profile to each ship.

5. Computational results

In this section we first illustrate how realistic test instances have been generated, then we present results obtained through a general-purpose solver and we compare them with our heuristic algorithm.

5.1. Generation of test instances

Our tests are based on real data provided by MCT. We had access to historical berth allocation plans and quay cranes assignment plans concerning about 60 vessels per week over a time horizon of 1 month; specific information on vessels such as the arrival time and the total number of containers to be handled were also provided. Furthermore, data referring to the flows of containers exchanged between ships as well as a study on the yard-related transshipment costs were available.

Instances generated to validate our models rely on these real data. The quay, which is 3395 m long, is partitioned in 13 berthing points, which are equipped with 25 quay cranes (22 gantry cranes and 3 mobile cranes). The matrix of distances $[d_{kw}]$ is a 13×13 matrix which takes into account the costs estimated by the terminal to move containers between two berthing positions. Several matrices of flows $[f_{ij}]$ are generated according to the distributions of containers reported in the historical data. As usual, we distinguish between feeders and mother vessels: the traffic volume is mostly influenced by the proportion between these two classes, since mother vessels present a number of loading/unloading containers on average higher than feeders. Time windows for the ships' arrival are generated according to the historical data. Berths are assumed to be available for the whole time horizon, which we set to 1 or 2 weeks. A working day is divided in 4 shifts of 6 h each, for a total of 56 time steps of 3 h.

The sets of feasible profiles have been synthetically generated in accordance with operational rules and good practices in use at the MCT terminal. As illustrated in Table 1, we fix a set of parameters for each ship class to which a profile must comply with in order to be feasible: namely, the minimum and the maximum number of QCs to be assigned to each vessel per shift as well as the minimum and the maximum handling time (HT) allowed for each class. We use a crane productivity of 24 containers per hour and we, therefore, obtain, per each class, a minimum and a maximum number of containers (column "volume" in the table): vessels' traffic volumes must comply with these ranges, according to the class they belong to. Furthermore, for all classes, a variation of at most 1 QC is allowed between a shift and the subsequent; profiles can start either at the beginning of the shift or in the middle of the shift.

Once the whole feasible set has been generated for each class, profiles are assigned to vessels according to the QC hours they need to be operated. At this point, a monetary value is associated to the couple (vessel, profile) with respect to the number of containers to be handled. This value is then adjusted by taking into account the profile's length and the utilized resources with respect to the average case.

To validate our model, we considered six classes of instances:

- 10 ships and 3 berths, 1 week, 8 quay cranes;
- 20 ships and 5 berths, 1 week, 13 quay cranes;
- 30 ships and 5 berths, 1 week, 13 quay cranes;
- 40 ships and 5 berths, 2 weeks, 13 quay cranes;
- 50 ships and 8 berths, 2 weeks, 13 quay cranes;
- 60 ships and 13 berths, 2 weeks, 13 quay cranes.

For each class, we generated 12 instances, with high (H) and low (L) traffic volumes. Each scenario is tested with a set of $\bar{p} = 10, 20, 30$ feasible profiles for each ship. We remark that, by construction, instances of size $\bar{p} = 10$ are included in instances of size $\bar{p} = 20$, which are included in instances of size $\bar{p} = 30$. Thus, any feasible solution for $\bar{p} = 10$ is also feasible for $\bar{p} = 20, 30$ and so on.

Table 1
Parameters for the profile set's generation.

Class	min QC	max QC	min HT	max HT	volume (min,max)
Mother	3	5	3	6	(1296, 4320)
Feeder	1	3	2	4	(288, 1728)

5.2. CPLEX computational results

The MIQP and MILP formulations have been tested with CPLEX 10.2, with emphasis on the feasibility of the solution.

Time limit for instances 10×3 is 1 h; instances 20×5 and 30×5 have a time limit of 2 h; instances 40×5, 50×8, 60×13 have a time limit of 3 h.

Results are illustrated in Table 2. We report only instances for which CPLEX has found a feasible solution, at least. Surprisingly, no feasible solution was found for classes 30×5, 50×8 and 60×13; however, an upper bound is always provided.

The objective function value is scaled to 100 with respect to the upper bound via the formula:

$$\text{scaled obj} = \frac{\text{obj} * 100}{\text{UB}} \tag{35}$$

A value of 100 means that the solution is certified to be optimal.

Within class 10×3, 3 out of 12 instances are solved at optimum; both MILP and MIQP formulations provide near-optimal solutions, with an average of 98.44 and 99.11, respectively.

Within class 20×5, a feasible solution is found for 4 instances out of 12 with the MILP formulation, while, using the MIQP formulation, we get a feasible solution only for two instances. The quality of the solution is lower, with an average of 93.87 for MILP and of 96.70 for MIQP.

Class 40×5 is only solved using the MILP formulation; a feasible solution is found for 4 instances out of 12, with an average quality of the solution of 94.73.

With respect to the upper bounds, we remark that the MILP formulation provides far better upper bounds than MIQP, as illustrated in Table 3.

5.3. Heuristic's computational results

The heuristic has been implemented in C++ using GLPK 4.31 and tested on the same set of instances.

Experiments have been run for $n \times \bar{p}$ iterations and a time limit of 1 h for classes 10×3, 20×5, 30×5 and 3 h for classes 40×5, 50×8, 60×13. The internal tabu search has a maximum of $\eta = 30 \times n$ iterations, and the other parameters are set as follows:

Table 2
Scaled objective function of the best feasible solutions found by CPLEX in the allowed time limit.

Instance	MILP	MIQP	Instance	MILP	MIQP
10 × 3			*10 × 3*		
H1_10	99.17	98.90	L1_10	97.68	100.00
H1_20	97.91	97.96	L1_20	100.00	99.76
H1_30	97.98	98.76	L1_30	98.64	99.99
H2_10	98.87	99.26	L2_10	98.82	99.63
H2_20	96.97	96.91	L2_20	99.42	99.06
H2_30	96.79	–	L2_30	99.08	100.00
20 × 5			*40 × 5*		
H1_10	94.33	–	L1_10	94.92	–
H1_20	93.74	–	L1_20	94.47	–
H2_10	93.52	96.66	L2_20	94.93	–
L2_10	93.87	96.74	L2_30	94.61	–

Table 3
Upper bounds provided by CPLEX using MILP and MIQP formulations.

Instance	MILP UB	MIQP UB	Instance	MILP UB	MIQP UB
30 × 5			*60 × 13*		
H1_10	1 754 291	2 288 451	H1_10	3 227 542	5 939 357
H1_20	1 754 633	2 288 793	H1_20	3 228 422	6 038 925
H1_30	1 754 669	2 288 829	H1_30	3 228 709	5 941 943
H2_10	1 708 485	2 256 299	H2_10	3 130 833	5 965 539
H2_20	1 709 020	2 256 834	H2_20	3 131 431	5 966 137
H2_30	1 709 230	2 257 044	H2_30	3 131 677	5 966 383
L1_10	1 420 485	1 787 983	L1_10	3 014 276	5 668 646
L1_20	1 420 713	1 817 824	L1_20	3 014 877	5 669 247
L1_30	1 420 819	1 842 700	L1_30	3 015 054	5 669 424
L2_10	1 613 252	1 948 130	L2_10	3 084 415	5 749 854
L2_20	1 613 769	1 973 914	L2_20	3 085 121	5 750 560
L2_30	1 613 805	2 008 053	L2_30	3 085 364	5 750 803

- θ: tabu duration equal to $\lfloor 7.5 \log n \rfloor$;
- β: diversification intensity parameter equal to $0.015\sqrt{nm}$;
- δ: penalty adjustment parameter equal to 2.

Results are compared to the best solution found by CPLEX for either the MILP or MIQP formulation and illustrated in Tables 4–6. The heuristic is able to find feasible solutions in 70 out of 72 instances, whereas CPLEX succeeds at finding feasible solutions on only 20 of the smaller instances. The two instances where the heuristic fails at finding a feasible solution are

Table 4
Heuristic's computational results on classes 10×3 and 20×5.

Instance	CPLEX	HEUR	Time (sec)	Instance	CPLEX	HEUR	Time (sec)
10 × 3				*20 × 5*			
H1_10	99.17	98.52	7	H1_10	–	97.26	81
H1_20	97.96	98.36	15	H1_20	94.33	97.19	172
H1_30	98.76	98.33	27	H1_30	93.74	97.37	259
H2_10	99.26	98.92	7	H2_10	–	97.27	82
H2_20	96.97	98.48	16	H2_20	96.66	97.38	173
H2_30	96.79	98.17	28	H2_30	–	97.26	274
L1_10	100.00	99.12	6	L1_10	–	97.30	74
L1_20	100.00	99.01	15	L1_20	–	97.25	158
L1_30	99.99	98.29	26	L1_30	–	97.06	254
L2_10	99.63	98.92	6	L2_10	–	97.55	80
L2_20	99.42	98.68	15	L2_20	96.74	97.39	170
L2_30	100.00	98.22	27	L2_30	–	97.25	295

Table 5
Heuristic's computational results on classes 30×5 and 40×5.

Instance	CPLEX	HEUR	Time (sec)	Instance	CPLEX	HEUR	Time (sec)
30 × 5				*40 × 5*			
H1_10	–	95.67	340	H1_10	–	97.38	1104
H1_20	–	95.31	677	H1_20	–	97.38	2234
H1_30	–	95.54	1009	H1_30	–	97.25	3387
H2_10	–	95.88	316	H2_10	–	97.40	1095
H2_20	–	95.81	684	H2_20	–	97.33	2198
H2_30	–	95.30	969	H2_30	–	97.27	3296
L1_10	–	96.55	324	L1_10	94.92	97.41	1421
L1_20	–	96.43	652	L1_20	94.47	97.14	2996
L1_30	–	96.18	966	L1_30	–	96.20	4862
L2_10	–	95.68	308	L2_10	–	97.41	1382
L2_20	–	95.12	614	L2_20	94.93	97.34	3144
L2_30	–	–	920	L2_30	94.61	96.60	4352

Table 6
Heuristic's computational results on classes 50×8 and 60×13.

Instance	CPLEX	HEUR	Time (sec)	Instance	CPLEX	HEUR	Time (sec)
50 × 8				*60 × 13*			
H1_10	–	96.52	3291	H1_10	–	95.40	6332
H1_20	–	96.37	6020	H1_20	–	95.07	10809
H1_30	–	96.21	9432	H1_30	–	94.76	10807
H2_10	–	96.03	3066	H2_10	–	95.54	6397
H2_20	–	95.64	6180	H2_20	–	94.11	10803
H2_30	–	95.16	9501	H2_30	–	–	10806
L1_10	–	95.97	2752	L1_10	–	95.67	5807
L1_20	–	96.04	6467	L1_20	–	95.40	10803
L1_30	–	95.80	9119	L1_30	–	94.45	10806
L2_10	–	96.18	3157	L2_10	–	95.63	5986
L2_20	–	95.96	5857	L2_20	–	95.64	10809
L2_30	–	96.27	8783	L2_30	–	95.34	10804

characterized by a high number of profiles per vessel ($\bar{p} = 30$). We observe that with a lower number of profiles per vessel ($\bar{p} = 10$, and $\bar{p} = 20$) the heuristic always succeeds in reaching feasibility. Furthermore, our algorithm is up to two orders of magnitude faster, especially on small instances.

Class 10×3 is the only one where CPLEX performs slightly better than the heuristic, with an average of 99.00 and 98.59, respectively, and three optimums found by CPLEX. However, the heuristic is much faster, solving the problem in less than 30 s against the time limit of 1 h set for CPLEX.

Class 20×5 is always solved by the heuristic in less then 5 min, with an average quality of the solution of 97.29, while CPLEX only solves four instances out of 12, in 2 h, with lower quality (95.37 on average).

Remarkably, our heuristic performs very well also on the instances of larger size, where CPLEX generally fails. For the solved instances the quality of the solutions is always greater than 94.11 (instance 60×13:H2_20), with an average value of 96.06.

6. Conclusions and future work

We have studied the integration, at the tactical level, of the berth allocation problem with the assignment of quay cranes from the point of view of a container terminal, in the context of a negotiation process with shipping lines. We have characterized this new decision problem and illustrated the concept of QC assignment profiles. Two mixed integer programming formulations have been presented, with a quadratic and a linearized objective function, respectively. Both models have been validated on instances based on real data using a commercial solver. These tests show that the problem is hardly solvable already on small instances; hence we have tackled the computational complexity of TBAP by devising a two-level heuristic algorithm able to provide good feasible solutions in a reasonable amount of time.

As a next step, we are interested in obtaining good upper bounds on the optimal solution. Decomposition methods seem to be a promising way to face the problem. In fact, we are considering a reformulation based on Dantzig–Wolfe decomposition and column generation, and an incremental approach based on Lagrangian dual, in order to exploit the structure of TBAP and its relation with the BAP formulation, aiming at saving computational time by solving subproblems via inexact or truncated methods.

With respect to the application, we remark that the main contribution is represented by the simultaneous control of the terminal on critical resources such as berths and quay cranes, in addition to the added value given by the integration, in a more direct way, of different terminal costs.

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