Part feeding and internal transportation decision-making for a machinery manufacturer

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Abstract

Due to a constant increase in the variety and number of parts required for assembly, part provision to the border of line requires careful orchestration. blackblueTherefore, every part must be assigned to the most suitable feeding policy, determining logistical in-house processes, pre-sorting degree, and load carrier sizing. This assignment allows effective space management at the border of line while avoiding unnecessary intra-logistics activities. Furthermore, timely part delivery necessitates decisions on vehicle type selection and delivery frequencies. In this research, we adapt an existing optimization model to the specific case of a machinery manufacturing company, validating the model for practical usage. The model aims to achieve cost minimization through optimal part feeding policy and vehicle type selection decisions, considering various constraints related to space availability at the border of line and the capacity of transportation vehicles. Our results reveal that the optimal assignment reduces costs by 56% compared with the company's current assignment. While the optimal assignment requires additional capacity investments for the company, we also test various restrictive solutions by adding practically relevant constraints. The latter still results in minimum cost savings of 23%, compared to the current assignment. blackblueBesides the cost benefits, the number of different line feeding policies is reduced. This streamlines the process, making it more conducive to automation.

1 Introduction

In an era where mass customization is becoming increasingly dominant due to individuals' preferences, many parts are requisite to produce a high variety of end products meeting these demands (Abdinnour, 2011; Adenipekun et al., 2022). The company involved in

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this research is a manufacturing company that assembles machinery on mixed-model assembly lines. The company produces many different product models on two pre-assembly and two final assembly lines. Feeding these assembly lines with the required parts is challenging and crucial for smooth production since more than 30 pre-assembly variants are produced within a single month and further customized downstream in the production process. Over the years, the company determined many different part feeding flows to timely feed the parts to the workstations, to be able to stay within the space constraints at the border of line, i.e., the area next to each assembly station designated for part storage, and given the available capacity of the logistics processes. Some of the flows involve preparation activities (such as presorting and repacking), and various vehicles, manual or automated, transport the bins from the warehouses to the workstations, with sometimes intermediate stocking points at logistical zones. Making the right choice between the myriad flows for every individual part is a difficult task. On the one hand, the impact of line-feeding decisions on performance is not easily quantified as part decisions are mutually dependent and affect intralogistics costs, delivery lead times, and space and process capacity requirements. On the other hand, it is of utmost importance that the line feeding system is reliable to deliver the right parts at the right time at the right location, avoiding line stoppage or ad hoc urgent deliveries.

The various process organizations defining how parts are packaged and routed through the factory are known as feeding policies. Twelve feeding policies are considered in this study, namely: line stocking, boxed supply (small, medium, large), sequencing, box picking, pallet picking, (type 1-3) stationary kitting, and (type 1 and 2) traveling kitting. These feeding policies, excluding box picking and pallet picking, stem from the five known feeding policies in literature, including line stocking, boxed supply, sequencing, stationary kitting, and traveling kitting. Feeding processes that determine the feeding costs comprise part replenishment, preparation, transportation, and usage at the assembly stations. The execution of these feeding processes is distinct for each feeding policy. Figure 1 depicts an overview of these processes. Every blackblueline-stocked part, packaged in pallet, is transported directly to the blackblueborder of line from the pallet warehouse. Line stocking remains the only feeding policy that does not require the preprocessing of parts. Parts assigned to every other feeding policy require repackaging at the preparation areas or the supermarket. In boxed supply (small, medium, or large), parts are repacked into homogeneously filled boxes at any of the preparation areas and delivered to the blackblueborder of line. In sequencing, functionally equivalent variant parts are repacked into sequenced containers according to the order of demand at the pallet preparation area, also known as the sequencing point (Swaminathan and Nitsch, 2007), and afterward delivered to the blackblueborder of line. In box picking or pallet picking, various parts required in the assembly stations within a time frame (usually a day) are repacked into pallets at the preparation areas and delivered to the logistical zone. From there, a logistics operator manually distributes parts to each assembly station. In kitting, diverse parts required for an end product are repacked into kits at the preparation area or supermarket and delivered to the assembly stations (type 1, type 2, or type 3 stationary kitting) or the start of the assembly line (type 1 or type 2 traveling kitting). blackblueWhile the line feeding policies section provides a concrete description of each of the feeding policies, the shop floor configuration section gives an additional explanation of the function of each specific facility in Figure 1, e.g., box warehouse and pallet warehouse.

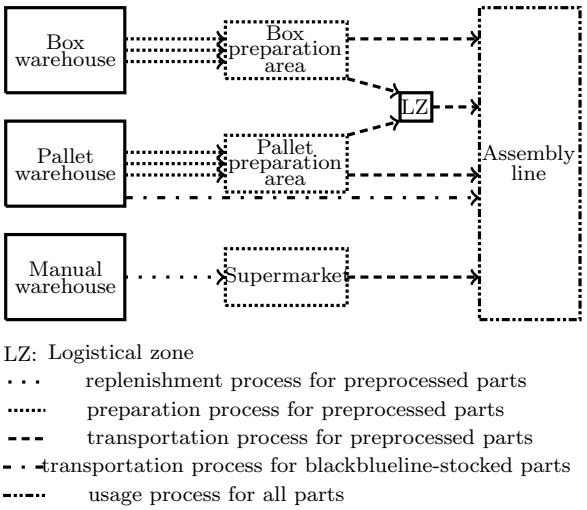


Figure 1: An overview of the four processes involved in assembly line feeding problems

In this study, part replenishment concerns refilling the supermarket with palletized parts from the manual warehouse (see the loosely dotted line in Figure 1). The preparation process takes place (*i*) in a picker-to-part manner at the supermarket or in a part-to-picker manner at the (*ii*) box or (*iii*) pallet preparation area. At the supermarket, a logistics operator picks stationary kits of type 3. Meanwhile, in the box or pallet preparation area, the preparation process involves waiting, as demonstrated by the logistics operator when boxes or pallets are conveyed by the conveyor to the respective preparation area. Additionally, part repackaging into policy-specific load carriers occurs at this preparation area (see the densely dotted lines in Figure 1). Part transportation is required regardless of the feeding policy considered. For instance, blackblueline-stocked parts are transported directly from the pallet warehouse to the assembly line (see the loosely dash-dotted line in Figure 1). Part transportation for other policy-specific parts occurs between any of the preparation areas or supermarket and the assembly line (see the dashed lines in Figure 1), with box and pallet picking parts temporarily stored at a logistical zone. The usage process entails the picking of parts at the blackblueborder of line by the assembly operator (see the densely dash-dot-dotted rectangle in Figure 1).

Assembly line feeding (ALF) models, specialized in tackling this problem, involve optimal individual part assignments to feeding policies to achieve feeding cost minimization (Schmid and Limère, 2019). The field of ALF remains nascent, and models are not adequate yet to easily allow for uptake by industry. In this research, we extend the model of Adenipekun et al. (2022) to make it applicable to the case study features. We have selected this model because it integrates assembly line feeding decisions with vehicle-type selection decisions. While vehicle types can be preassigned to feeding policies, integrating this decision increases the degrees of freedom to optimize the logistics flows. In addition, within the model, parts' delivery frequency is aligned with the vehicles' capacity and the storage space at the border of line to maintain adequate inventory levels. Although the model of Adenipekun et al. (2022) is sufficient for the problem statement of the company, the particularities of the company demanded an extension of the model with regards to the following aspects: (*i*) while the scope of the previous work included one warehouse and one supermarket (i.e., the area for parts preparation), the current research comprises two automated warehouses and a manual warehouse, where each warehouse has its respective preparation area or supermarket; (*ii*) picking of parts during the preparation

process was modeled in a picker-to-parts manner, however, the company's preparation process for parts initially stored at any of the automated warehouses is done in a parts-to-picker manner (see the process description section for more details); (*iii*) the modeling approach in the previous work included five feeding policies, whereas the modeling approach in the current study extends those line feeding policies to incorporate the twelve feeding policies enumerated earlier; (*iv*) whereas the modeling approach was validated by artificial problem instances, now validation is done based on assembly data provided by the company. The approach is applied to a mixed-model pre-assembly line of seven workstations, performing assembly tasks on 36 products and involving 158 parts.

The remainder of this paper is structured as follows: We described the state of the art in the literature review section. Within the process description section, the company-specific processes, including the shop floor configuration and line feeding policies, are extensively described. While the modeling section discusses the adaptations and extensions of the model, we present the complete model in the mathematical model and cost parameter calculation sections of the appendices. Next, the analysis of the current and optimal solution section compares the company's current decisions to the optimal solution obtained by the model. Furthermore, in the multi-case analysis section, we test various more constrained versions of the model arising from our discussions with the decision-makers. The conclusion and recommendation section provides a summary, limitations, and possible opportunities for future research.

2 Literature Review

In this section, we review the literature focusing on the main decisions in our approach. We consecutively discuss research on assembly line feeding, vehicle type selection, and the integration between both.

2.1 Assembly Line Feeding

Bozer and McGinnis (1992) pioneered the research in applying process costing to different feeding policies, namely line stocking and stationary kitting. Factors such as space requirements, material handling efforts, and work in process are considered in the cost calculations. Based on this, decision-makers could choose either of the two feeding policies for all the parts. This approach has been used and extended by many researchers in the field. Caputo and Pelagagge (2011) later built on the study by proposing an ABC method to group parts and assign them to feeding policies. Battini et al. (2010) combined the decision to either centralize or decentralize storage areas and assembly line feeding decisions in their study by minimizing feeding and storage costs when choosing between line stocking, stationary kitting, and traveling kitting. Later on, Sali et al. (2015) developed a descriptive model based on a case study in the automotive industry. Criteria such as part features, part family diversity, and blackbluebill of material coefficients were considered when selecting the cost-efficient feeding policy between line stocking, sequencing, and kitting.

As one may notice, research efforts aimed at determining the most efficient feeding policy for each individual part rather than a global level as initially proposed by Bozer and McGinnis (1992). Therefore, optimization-based models, determining a feeding policy for

each part among all feeding policies considered while minimizing the associated feeding costs, were introduced. Notably, Limère et al. (2012) developed a mathematical programming model that determines for each part whether it should be blackblueline-stocked or stationary-kitted. In addition, variable operator walking distances were included in an extension of this work (Limère et al., 2015). Sali and Sahin (2016) proposed an optimization model, based on a case study in the automotive industry, that decides the best feeding policy for every part. The feeding policies considered are line stocking, sequencing, and traveling kitting. According to the authors, available space at the blackblueborder of line and the kit container's capacity are the main trade-off to consider when deciding between these policies. Baller et al. (2020) also developed an optimization model that is applied to a case study at an automotive company. Furthermore, Schmid et al. (2020) included five line feeding policies into their optimization model and simultaneously incorporated discrete location decisions, i.e., the parts' placement at the border of line, into the modeling approach.

Recently, Moretti et al. (2021) and Zangaro et al. (2021) proposed supervised machine learning approaches for the assembly line feeding problem in order to provide practitioners with guidelines to follow when selecting feeding policies for every component. Moreover, Müllerklein et al. (2022) developed an optimization model that integrates assembly line feeding and scheduling problems. Following the application of the model to a case study, the result shows that the production schedule influences feeding policy decisions and the associated costs.

2.2 Vehicle Type Selection

The internal transportation process on the shop floor is vital as every part, irrespective of the feeding policy assigned, requires transportation. Battini et al. (2015) studied the transportation mode selection problem through a descriptive model that discusses some heuristics selection rules. According to the authors, transportation selection decision is mainly influenced by the distance covered during the transportation operation, the takt time, and the part container demand per takt. Nourmohammadi et al. (2021) developed an optimization model based on supermarket location and transport vehicle selection decisions. Results showed that using different vehicle types is more beneficial compared to identical vehicle type usage. Emde and Boysen (2012) proposed an optimization model that solves the routing and scheduling problem of tow trains between the supermarket and assembly stations. The model decides on the number of consecutive assembly stations required to be served by a tow train. In addition, parts are promptly delivered to the assembly stations as the model decides on the start times of the tow trains. In another study, Cheng et al. (2017) proposed an optimization model, simultaneously considering inventory routing problem and vehicle type selection, which minimizes inventory cost, driver wage, vehicle acquisition cost, and fuel and emission costs. According to the authors, the result shows that using a heterogeneous fleet is preferred to a homogeneous fleet in terms of cost savings. We remark that the scope of assembly line feeding decisions is not included in these studies.

2.3 Problem Integration

The study on integrating assembly line feeding and vehicle type selection decisions was recently put forward when Adenipekun et al. (2022) developed an optimization model for this purpose. Three vehicle types were considered namely: AGVs, forklifts, and tow trains. To quantify transportation costs more accurately, the transportation process is performed by selecting specific routes from a set of predefined routes. In addition, the fleet size of every vehicle type used was also determined. According to the authors, an optimal selection of vehicle types is superior to heuristic scenarios in terms of cost savings and feasibility. Before this time, Adenipekun et al. (2020) proposed a conceptual framework that integrates assembly line feeding and route building. Results showed that vehicle types capable of serving multiple stations at once are preferred.

While the previous works on integrating assembly line feeding and vehicle type selection decisions discuss benefits regarding accurate computation of transportation distances and overall cost-savings, these works are based on artificial problem instances (Adenipekun et al., 2022). Moreover, the ALF case studies in literature do not consider vehicle type selection decisions (Baller et al., 2020; Sali and Sahin, 2016). This study, however, validates an optimization model that integrates assembly line feeding and vehicle type selection decisions, by a case study. In addition, the model chooses a cost-efficient feeding policy for each part from the twelve feeding policies considered.

3 Process Description

This section describes the configuration of the shop floor and the processes involved in each of the line feeding policies in the case study considered.

3.1 Shop Floor Configuration

Before parts can be used, they are received from various suppliers and stored in one of the warehouses. In this research, we distinguish between three kinds of warehouse, namely: box warehouse, pallet warehouse, and manual warehouse. The box and pallet warehouses are automated warehouses. While parts in pallets are initially stored at the pallet warehouse, the box warehouse houses parts that are initially stored in boxes. These parts are brought to their respective preparation area (i.e., box and pallet preparation areas) by a conveyor in a part-to-picker manner. We define a conveyor in this case study as a system designed to move pallets or boxes from the automated warehouses to their respective preparation area. The repackaging of parts is performed at these preparation areas before parts are delivered by a designated vehicle to the blackblueborder of line (see Figure 2). By contrast, parts initially stored at the manual warehouse, usually in pallets, are brought to the supermarket by forklifts. At the supermarket, parts are repackaged into the dedicated load carriers by a logistics operator in a picker-to-part manner. Afterward, the transportation process is performed by delivering parts to the blackblueborder of line by a designated vehicle (see Figure 2). In this case study, we initially store parts assigned to type 3 stationary kitting in the manual warehouse. Line-stocked parts do not undergo the repackaging process since they are delivered directly to the blackblueborder of line from the pallet warehouse. Figure 2 provides the complete representation of the shop floor as described above. We remark that only the areas with

yellow backgrounds in the schematic representation are in the scope of this case study. Moreover, the red arrows represent the workflow across assembly stations. This research considers four vehicle types: AGVs, forklifts, tow trains, and manual carts or manual push-carts. AGVs, forklifts, and manual carts serve a single station at a time, whereas tow trains serve multiple stations in a milk-run manner along a predefined route. Regarding assembly stations, the space at the blackblueborder of line varies across stations, and the size of the blackblueborder of line in each station depends on the number of pallet spaces available in that station.

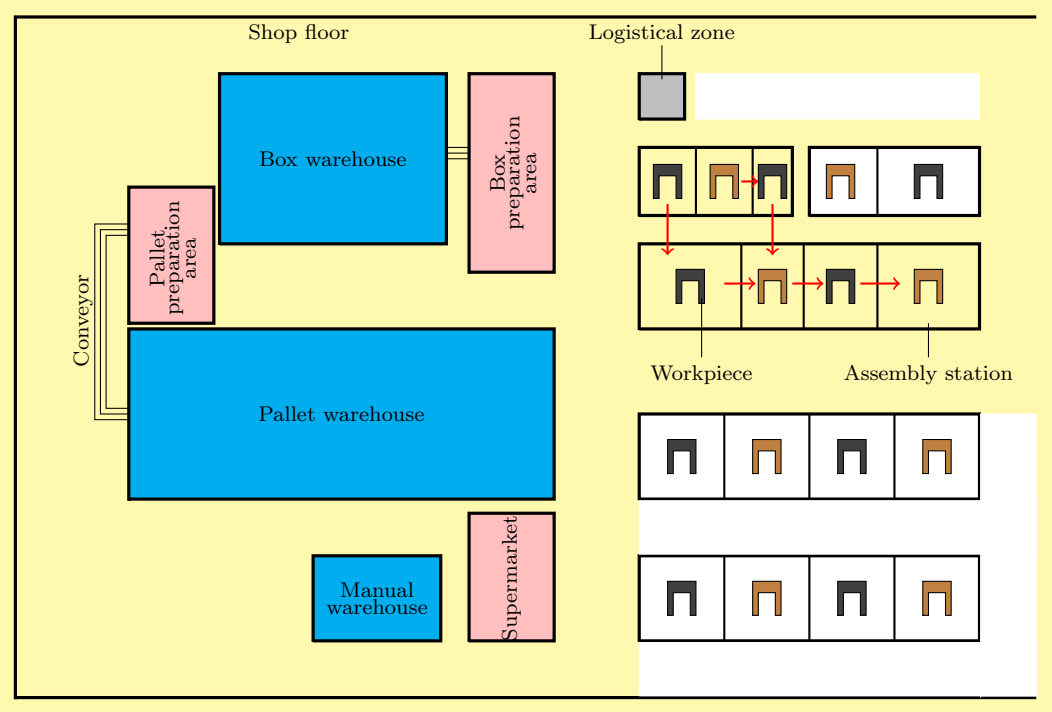


Figure 2: Shop floor schematic representation

3.2 Line Feeding Policies

In this study, we consider twelve feeding policies. Although the company provides parts to the assembly stations' blackblueborder of line through nine feeding policies, three feeding policies, i.e., sequencing and (type 1 and type 2) traveling kitting, have been included based on their occurrence in the literature (see Adenipekun et al. (2022); Schmid et al. (2020)) to investigate the potential benefits of these policies for the company under consideration. Furthermore, ten of the twelve feeding policies considered in this framework extend from the five feeding policies considered in Adenipekun et al. (2022). Box picking and pallet picking, the two additional feeding policies, have not been considered previously. Table 1 depicts an overview of the feeding policies alongside their associated processes. Below is the process description for each of these feeding policies.

In **line stocking**, parts are initially stored at the pallet warehouse (see Figure 2). From this point, the designated vehicle delivers parts directly to the blackblueborder of line. Hence, they do not require preprocessing. Line-stocked parts typically arrive in pallets, with each pallet containing a single SKU. From the box warehouse, parts assigned to **boxed supply (small and medium)** are brought to the box preparation area through a conveyor (see Figure 2). Afterward, the logistics operator repackages homogeneous

Table 1: Feeding policies and the associated processes

Line feeding policy	**Packaging form**	**Process**				**Assembly station**
		Repl.	**Prep.**	**Trans.**	**Usage**	
Line stocking	Single SKU	×	×	PW→BoL	✓	Single
Boxed supply (small)	Single SKU	×	✓	BPA→BoL	✓	Single
Boxed supply (medium)	Single SKU	×	✓	BPA→BoL	✓	Single
Boxed supply (large)	Single SKU	×	✓	PPA→BoL	✓	Single
Sequencing	Single PF	×	✓	PPA→BoL	✓	Single
Box picking	Multiple PF	×	✓	BPA→LZ→BoL	✓	Multiple
Pallet picking	Multiple PF	×	✓	PPA→LZ→BoL	✓	Multiple
Type 1 stationary kitting	Multiple PF	×	✓	BPA→BoL	✓	Single
Type 2 stationary kitting	Multiple PF	×	✓	PW→BoL	✓	Single
Type 3 stationary kitting	Multiple PF	✓	✓	S→BoL	✓	Single
Type 1 traveling kitting	Multiple PF	×	✓	BPA→AL	×	Multiple
Type 2 traveling kitting	Multiple PF	×	✓	PW→AL	×	Multiple

SKU: stock-keeping unit; PF: part family; BPA: box preparation area
PPA: pallet preparation area; PW: pallet warehouse; S: supermarket
LZ: logistical zone; BoL: border of line; AL: assembly line

parts into boxes, while the designated vehicle delivers the boxes to the blackblueborder of line. The difference between boxed supply (small and medium) lies in the size of the load carriers. From the pallet warehouse, parts assigned to **boxed supply (large)** are brought in pallets to the pallet preparation area through a conveyor and repackaged into boxes (see Figure 2). Subsequently, the designated vehicle delivers load carriers to the blackblueborder of line.

In **sequencing**, a logistics operator typically arranges variants of the same part family within a container according to the demand order. A part family is a group of functionally equivalent parts that generally differ in features like color or quality. In this case study, a conveyor moves parts assigned to sequencing from the pallet warehouse to the pallet preparation area. Subsequently, a logistics worker sequences the parts into the corresponding load carrier (see Figure 2), after which the dedicated vehicle delivers load carriers to the blackblueborder of line.

In **pallet and box picking**, a logistics operator generally arranges parts of all assembly stations considered on a pallet based on the demand at the stations during a predefined period, usually a day, while respecting the pallet's capacity, the logistics operator may use multiple pallets for the picking operation. In this case study, a conveyor moves the pallet-picked parts from the pallet warehouse to the pallet preparation area, where a logistics operator sorts parts onto pallets (see Figure 2). Upon completion of the preparation process, the designated vehicle transports the load carriers to the logistical zone, which is situated between the preparation area and the assembly stations, serving as a temporary storage area for pallet picking and box picking parts (see Figure 2). Afterward, an internal logistics worker manually distributes the parts to their respective stations from the logistical zone. The difference between box picking and pallet picking is that box-picked parts are initially stored in a box at the box warehouse and are repackaged onto a pallet, whereas the pallet warehouse houses the pallet-picked parts on pallets. Although the final provision to the blackblueborder of line is identical, the storage location and repacking process differ for those line feeding policies.

In kitting, parts from different part families required for an end product are sorted and placed in a carrier. In this study, parts assigned to **type 1 stationary kitting** are conveyed from the box warehouse to the box preparation area by the conveyor and are kitted into boxes. Upon completion of the preparation process, the conveyor moves the boxes back to the box warehouse. When needed at the assembly stations, the boxes are brought to the warehouse exit, placed on pallets, and transported by a dedicated vehicle to the blackblueborder of line (see Figure 2). Although type 1 stationary kitting and **type 1 traveling kitting** go through similar procedures, there is a distinction in their delivery process: a dedicated vehicle delivers type 1 stationary-kitted parts to different assembly stations at the blackblueborder of line, whereas parts assigned to type 1 traveling kitting are delivered directly to the assembly line. Parts assigned to **type 2 stationary and traveling kitting** are initially housed at the pallet warehouse, conveyed by the conveyor to the pallet preparation area, and kitted into pallets. Afterward, pallets are moved into the pallet warehouse and later brought to the warehouse exit through the conveyor. A dedicated vehicle, then, delivers pallets to the blackblueborder of line of different assembly stations (type 2 stationary kitting) or directly to the assembly line (type 2 traveling kitting). In **type 3 stationary kitting**, parts are initially stored in pallets at the manual warehouse. The replenishment process is performed by forklifts when pallets are delivered to the supermarket (see Figure 2). Afterward, parts are kitted into special carts and transported by tow trains to the blackblueborder of line.

Table 1 summarizes these feeding policies and their associated processes as follows: (*i*) the load carrier used for the feeding policies may contain single part (SKU or stock-keeping unit), single part family or multiple part family; (*ii*) replenishment process is only concerned with type 3 stationary kitting as every other feeding policy does not require replenishment; (*iii*) all feeding policies except line stocking requires repackaging; (*iv*) irrespective of the feeding policy, transportation operation must be performed. However, transportation flow may differ across feeding policies, e.g., transportation flow for line stocking is between the pallet warehouse and the blackblueborder of line, whereas boxed supply (small)'s transportation flow is between the box preparation area and the blackblueborder of line; (*v*) all feeding policies, except type 1 and type 2 traveling kitting, undergo the usage process as a dedicated vehicle delivers the load carriers for these policies to the blackblueborder of line during the transportation operation, and; (*vi*) parts may be used at single or multiple assembly stations, depending on the feeding policy, e.g., blackblueline-stocked parts are used at a dedicated station, by contrary, parts assigned to traveling kitting are used at multiple stations since the kits move along multiple stations with the end product.

4 Modeling

The model used throughout this study extends the model of Adenipekun et al. (2022). Given the specific demands of the company, this model differs from the previous one in the following aspects: the inclusion of multiple warehouses and the associated preparation area or supermarket; the preparation process for parts initially stored in any of the automated warehouses is done in a part-to-picker manner instead of picker-to-part (see the process description section); the inclusion of twelve feeding policies, and; model validation through the assembly data provided by the company. The model takes as input 158 parts,

with their corresponding attributes such as demand, volume, and weight. Furthermore, the planning horizon, the available space at the blackblueborder of line of each station, and the capacity of each vehicle type used also serve as input to the model. As an output, the model returns the assignment of every part to the most suitable feeding policy and vehicle type. The model minimizes overall feeding costs, consisting of operating and investment costs. The operating costs are concerned with costs accrued from several processes, i.e., replenishment, preparation, transportation, and usage process, whereas the investment costs comprise purchasing and maintenance costs of vehicles. We will present details about the model description and the cost drivers in the subsequent sections. Below are the assumptions used in the model:

- We assume part demand to be deterministic.
- The process-related operations such as preparation and transportation are assumed to have deterministic duration. Therefore, the usage of fixed parameters calculates the associated costs.
- For every end product, we assumed the usage of, at most, a stationary kit per station and a traveling kit.
- While manual carts may serve multiple stations in reality, each cart can only serve one station at once in this work.

4.1 Model Description

In the following, we will describe the model while presenting the mathematical model and cost calculations in the mathematical model and cost parameter calculation sections of the appendices, respectively. The model comprises the objective function, general constraints, route construction, line-sided storage, and fleet size determination. As stated at the beginning of this section, the objective function in (1) aims to minimize the costs related to the feeding process and the vehicles' acquisition and maintenance costs. The process consists of replenishment, preparation, transportation, and usage process. We will extensively discuss each process in the cost drivers section and present the cost parameters concerning these processes for associated feeding policies in the cost parameter calculation section of the appendices.

The general constraints comprise the restrictions concerning the assignment of every part to a single feeding policy and vehicle type, the number of pallets required for box-picked and pallet-picked parts, the usage of kits for every end product at each station (stationary kitting) or at the line (traveling kitting), and the definition of the domains of all variables used in the model (see Constraints (2) to (19) in the general constraints section of the appendices).

The route construction in this study is similar to the one proposed in Adenipekun et al. (2022). The authors defined a route as a path between the warehouse or supermarket and a consecutive sequence of stations at the assembly line. In this research, the route length is the same across all routes since the manufacturing company under consideration is a complete brownfield plant. Hence, every route must complete the entire milk-run length regardless of the number of stations in the route. We also consider the traveling

frequency in this framework. By so doing, when a tow train is assigned to deliver parts through a route, a suitable number of takts t, which corresponds to how frequently the tow train will serve the route during the planning horizon, is dedicated to such assignment. For instance, in a given route, $t = 2$, among set $\mathcal{T}$, signifies the assigned tow train will traverse the route every two takts during the planning horizon (see Constraints (20) and (21) in the route construction section of the appendices).

Ensuring a continuous flow of parts on the shop floor necessitates space management at the blackblueborder of line, which entails harnessing both the demand for parts and the frequency of their delivery to the blackblueborder of line. Generally, more space will be assigned to store parts with low frequency at the blackblueborder of line, whereas parts with high frequency often get less blackblueborder-of-line space. A *facing* is the space along the blackblueborder of line in each station whose size is enough to store a single part container of a corresponding feeding policy, e.g., the size of a pallet in line stocking. Constraints (22) to (29) in the line-sided storage section of the appendices ensure proper space management at the blackblueborder of line. In this case study, once a vehicle is assigned to perform the transportation operation of any part, we determine the vehicle fleet needed during the planning horizon. Constraints (30) to (39) in the fleet size determination section of the appendices present the constraints corresponding to each flow and the associated vehicle types.

4.2 Cost Drivers

Forklifts generally deliver parts from the warehouse to the supermarket during part replenishment in the company under consideration. In this study, part replenishment is only concerned with parts assigned to type 3 stationary kitting, as other parts that require preprocessing are initially stored at the pallet or box warehouse and delivered to their respective preparation area through conveyors. Replenishment time is the time needed to perform the replenishment process. The replenishment cost refers to the product of the hourly forklifts' operating cost and the replenishment time. Equation (41) in the replenishment cost section of the appendices describes the replenishment cost for type 3 stationary kitting.

The preparation process entails parts repacking into policy-specific load carriers in one of the preparation areas. For instance, the logistics operator repackages boxed supply (small) parts into boxes. As discussed earlier, parts initially stored in automated warehouses (either box or pallet) are moved to the corresponding preparation area through conveyors in a part-to-picker manner. During this time, the logistic operator at the preparation area must wait for part arrival from the warehouse. Hence, the preparation cost for parts initially stored in automated warehouses factors in the logistics operator's waiting time. The preparation time refers to the time required to complete the preparation process. Ultimately, the preparation cost is the product of hourly logistics operator cost and the preparation time. The preparation cost for all feeding policies requiring preprocessing is presented in Equations (42) to (45) in the preparation cost section of the appendices.

Upon completion of the preparation process, designated vehicles must transport parts from their respective preparation area or supermarket to assembly stations. For parts that are not prepared in any way, i.e., blackblueline-stocked parts, also require transporta-

tion, i.e., from the pallet warehouse to the corresponding assembly stations. However, transportation times for parts are policy-dependent and vehicle-dependent since the load carriers used for each feeding policy are generally unique, and the capacities of the vehicles vary. As described above, AGVs, forklifts, and manual carts only serve a single station, whereas tow trains may serve multiple stations at once in a milk-run manner. The transportation time entails the required time necessary to complete the transportation process. The transportation cost is the product of a vehicle type's operating cost per hour and the transportation time. Equations (46) to (60) in the transportation cost section of the appendices present the transportation cost associated with every feeding policy and vehicle type considered.

The usage process mainly describes walking between the assembly line where the assembly operator works on the end product and the blackblueborder of line where parts are stored. In addition, some searching may be needed, depending on the feeding policy. The usage time refers to the time required to perform the usage process. Therefore, the usage cost is the product of an assembly operator's hourly wage and the usage time. Equations (61) to (62) in the usage cost section of the appendices present the usage costs of all feeding policies undergoing the usage process). Parts assigned to (type 1 and type 2) traveling kitting bypass the usage process since these kitted parts are delivered to the assembly line directly.

Within this case study, the input data consists of seven stations, 158 parts, 149 part families, and the information given in Table 13. The set of takt numbers, which decides the frequency degree of tow trains during the planning horizon, is
$\mathcal{T} = \{1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360\}$. Furthermore, the number of end products is 36. We received all input data used in this study from the manufacturing company. The maximum number of variables and constraints over all cases in the model are $18,216$ ($10,364$ binary) and $29,218$, respectively.

5 Analysis of the Current and Optimal Solution

The model is solved optimally using Gurobi 9 on a computer with 8 GB RAM and an Intel Core i5-8250 processor with 1.8 GHz. The optimal assignment only required about 600 seconds to be found. In this section, we compare the results obtained from the optimal solution to the current situation at the manufacturing company. This is presented in the cost component analysis, feeding policy assignment, vehicle type assignment, and space usage at the blackblueborder of line sections. Furthermore, in the multi-case analysis section, we will analyze multiple cases requested by the company to gain further insights.

5.1 Cost Component Analysis

As a result of the variation in part assignment to feeding policy and vehicle type in the current situation and optimal solution (see the feeding policy assignment and vehicle type assignment sections), the observed changes in the cost components are explained as follows. The total feeding and acquisition costs comprise the replenishment, preparation, transportation, usage, and acquisition costs. A total cost reduction of €2509.18 is monthly realized when comparing the feeding and acquisition costs in the current assignment with the optimal assignment, i.e., from €4470.76 per month in the current

assignment to €1961.58 per month in the optimal assignment. As shown in Figure 3, we will explain the realization of the cost reduction.

The replenishment cost incurred in the current assignment is due to the parts assigned to type 3 stationary kitting, as this is the only policy requiring part replenishment. By contrast, type 3 stationary kitting is dormant in the optimal solution. The cost reduction in part preparation when comparing the optimal assignment to the current assignment is explained as follows: (*i*) more parts are assigned to line stocking in the optimal solution, not requiring any preparation. The use of (type 1 and type 2) traveling kitting in the optimal solution facilitates this assignment, releasing more space at the blackblueborder of line for the blackblueline-stocked parts; (*ii*) the usage of type 3 stationary kitting used in the current assignment does not occur in the optimal assignment. Recall that parts assigned to type 3 stationary kitting are prepared in a picker-to-part manner (see the process description section), which in turn leads to high preparation cost and; (*iii*) the reduction is mainly due to the shifts in utilizing less boxed supply (small) and more boxed supply (medium) in the optimal assignment compared to the current assignment. Notice that the waiting time required to prepare the containers for boxed supply (small and medium) is the same. However, the latter's container is larger than the former.

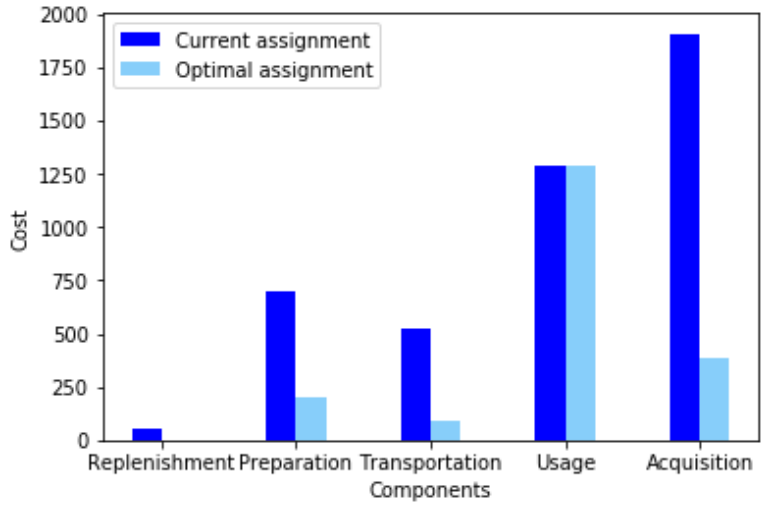


Figure 3: Cost components over current and optimal assignments

The optimal solution also comprises a much lower transportation cost than the current assignment. The main factor leading to this change is the utilization of type 3 stationary kitting in the current assignment. Recall that tow trains deliver parts assigned to this feeding policy on the chosen route. According to the result, a tow train transports parts to assembly stations every takt, resulting in high transportation costs. On the contrary, the optimal solution avoids type 3 stationary kitting. Elsewhere, in the optimal assignment, parts assigned to line stocking and type 2 traveling kitting are delivered to the blackblueborder of line by an AGV. We remark that an AGV has no operating cost. Every part in the current assignment and approximately 93% of the parts in the optimal assignment undergo the usage process. Therefore, usage cost is almost equal across the two scenarios. The fact that parts assigned to traveling kitting do not require the usage process accounts for the slight difference. Hence, these parts avoid the usage cost. Finally, the acquisition cost further impacts the overall costs, as shown in Figure 3. While delivery of parts to the blackblueborder of line in the optimal assignment is either

performed by manual carts or AGV, a tow train is among the vehicle types in the current assignment delivering parts to the blackblueborder of line. Hence, the vehicle types used in the optimal assignment are more utilized when compared with the current assignment. In addition, the acquisition cost in the current assignment also includes forklifts when delivering stationary-kitted parts during the replenishment process.

5.2 Feeding Policy Assignment

This section presents the changes observed in feeding policy assignment to parts across the current and optimal assignment. Table 2 delineates the distribution of parts over the feeding policies considered. The rows represent part distribution based on the current assignment, whereas the column depicts part distribution according to the optimal assignment. Consequently, we can seamlessly identify shifts in feeding policy assignments from the current to the optimal assignment. The overall cost savings realized in the optimal solution and other factors discussed in the sequel characterize these shifts. The number of parts assigned to line stocking increased from 10 in the current assignment to 24 in the optimal assignment. The introduction of traveling kitting in the optimal assignment where space at the blackblueborder of line is not required to store traveling-kitted parts accounts for this increment, thereby providing more blackblueborder-of-line space for parts assigned to other feeding policies, e.g., line stocking. Furthermore, in the optimal assignment, while the average part demand for blackblueline-stocked parts increases by 35% of the corresponding average part demand in the current assignment, the average volume decreases by 45%. Therefore, blackblueline-stocked parts in the optimal assignment have lower volumes when compared with the current assignment.

The number of parts assigned to boxed supply (small) reduces drastically in the optimal assignment. Boxed supply (small) is not an enticing choice as far as the optimal assignment is concerned (see the cost component analysis section). Although the average volume of parts assigned to boxed supply (small) across the current and optimal assignment remains the same, the average demand in the current assignment reduces by 31%. Consequently, other feeding policies like boxed supply (medium, large) and type 1 traveling kitting, benefiting from this change in assignment, are of higher average demand or volume.

Although the optimal assignment highly favors boxed supply (medium) due to the advantage it has over boxed supply (small), most boxed supply (medium) parts in the current assignment prefer other feeding policies such as line stocking and boxed supply (large) in the optimal assignment. Besides the additional preprocessing cost unrequired in line stocking, parts assigned to line stocking and boxed supply (large) in the optimal assignment are of higher volume when compared with boxed supply (medium). Although in the optimal assignment, boxed supply (large) mainly benefits from boxed supply (small and medium) parts due to the reasons given above, none of the boxed-supply (large) parts in the current assignment appear in the optimal assignment. These parts are assigned to type 1 traveling kitting or line stocking due to the volume requirement of the parts and possibly the blackblueborder-of-line space that is unrequired for traveling kitted parts.

Even though sequencing is unused in the current assignment, we included the feeding policy in this case study to exploit the gains. However, we noticed that none of the parts

prefer sequencing in the optimal assignment. This result is unsurprising as sequencing flourishes when there are multiple-variants part families. Contrarily, approximately 95% of the part families are single-variant. Parts provided to the blackblueborder of line through box picking, pallet picking, type 1 and type 2 stationary kitting in the current situation seem to prefer other feeding policies in the optimal solution (see Table 2). In type 3 stationary kitting, none of the parts in the current assignment retain the same feeding policy in the optimal assignment. The parts in this category, which are generally high in volume and weight, prefer feeding policies, such as line stocking, type 2 traveling kitting, and box picking. Generally, blackblueline-stocked parts are in high demand, type 2 traveling kitted parts are in medium-to-low demand, whereas the parts assigned to box picking are in low demand.

Table 2: Part distribution based on feeding policies in the current and optimal assignment

from \ to	**LS (24)**	**BSS (45)**	**BSM (47)**	**BSL (27)**	**SEQ (0)**	**BP (4)**	**PP (0)**	**T1S (0)**	**T2S (0)**	**T3S (0)**	**T1T (7)**	**T2T (4)**
LS (10)	9	-	-	-	-	-	-	-	-	-	1	-
BSS (106)	-	43	40	19	-	-	-	-	-	-	4	-
BSM (18)	7	1	4	6	-	-	-	-	-	-	-	-
BSL (3)	1	-	-	-	-	-	-	-	-	-	2	-
SEQ (0)	-	-	-	-	-	-	-	-	-	-	-	-
BP (2)	-	-	2	-	-	-	-	-	-	-	-	-
PP (4)	-	-	1	1	-	2	-	-	-	-	-	-
T1S (2)	-	1	-	1	-	-	-	-	-	-	-	-
T2S (2)	2	-	-	-	-	-	-	-	-	-	-	-
T3S (11)	5	-	-	-	-	2	-	-	-	-	-	4
T1T (0)	-	-	-	-	-	-	-	-	-	-	-	-
T2T (0)	-	-	-	-	-	-	-	-	-	-	-	-

LS: Line stocking, BSS: Boxed supply (small), BSM: Boxed supply (medium)
BSL: Boxed supply (large), SEQ: Sequencing, BP: Box picking, PP: Pallet picking
T1S: Type 1 stationary kitting, T2S: Type 2 stationary kitting, T3S: Type 3 stationary kitting
T1T: Type 1 traveling kitting, T2T: Type 2 traveling kitting

5.3 Vehicle Type Assignment

Not only are the changes observed in part assignment, across the current and the optimal assignment, limited to feeding policies, but they also concern vehicle types. Table 3 depicts the distribution of parts over the vehicle types considered. On the one hand, the rows represent part distribution based on the current assignment. The columns, on the other hand, show part distribution concerning the optimal assignment. Eleven of the 14 parts initially delivered to the blackblueborder of line by AGVs still utilize AGVs in the optimal assignment as AGVs are generally known for pallets or kit-containers transportation. Manual carts deliver the remaining three parts to the blackblueborder of line. For parts transported by tow train to the blackblueborder of line in the current assignment, AGVs and manual carts transport 50%, each, in the optimal assignment. Manual carts deliver 124 parts to the blackblueborder of line in the current assignment. By contrast, AGVs, forklifts, and manual carts transport seven, three, and 114 parts to the blackblueborder of line, respectively, in the optimal assignment.

Generally, we noticed AGVs deliver parts that are high in weight to the blackblueborder of line. By contrast, manual carts transport lower-weight parts to the blackblueborder of line. Furthermore, it is worth mentioning that AGVs and manual carts are enticing vehicle types among the vehicle types considered since AGVs do not accrue operational

costs, i.e., the only cost attached to AGVs is the acquisition cost. Manual carts, however, incur a low acquisition cost. In the optimal assignment, we notice the usage of AGVs for part transportation over a long distance, e.g., blackblueline-stocked and kitted parts. On the contrary, manual carts generally transport parts over short distances, e.g., boxed supply (low and medium) parts.

Table 3: Part distribution based on vehicle types in the current and optimal assignment

from \ to	AGV (28)	FL (3)	TT (0)	MC (127)
AGV (14)	11	-	-	3
FL (0)	-	-	-	-
TT (20)	10	-	-	10
MC (124)	7	3	-	114

AGV: Automated guided vehicle, FL: Forklift
TT: Tow train, MC: Manual cart

5.4 Space Usage at the blackblueBorder of Line

Space management is of utmost importance in assembly line feeding. Therefore, analyzing space usage at the blackblueborder of line becomes imperative. Tables 4 and 5 respectively show the usage of space and rack usage at the blackblueborder of line in the current and optimal assignment. Figures 4 and 5, which are the graphical representation for Tables 4 and 5, respectively, depict the percentage usage of pallet spaces and racks in the current and optimal assignment across the assembly stations. The available space in each station refers to the number of pallet spaces that can store parts accordingly, whereas the unused space is the number of pallet spaces that are unused throughout the planning horizon in each station. Notice that a pallet space can store a pallet, a kit container, or a rack housing containers for boxed supply (small, medium, large), box picking, and pallet picking. Therefore, at the blackblueborder of line of every station, the sum of the number of pallet space needed to store pallets for line stocking (LS), type 1 (T1S), type 2 (T2S), type 3 (T3S) stationary kit containers, and racks (Racks), subtracted from the available space (AS) yields the unused space (US). We remark that the number of racks (Racks) is an upper bound for the sum of the number of facings needed for boxed supply (small (BSS), medium (BSM), large (BSL)), box picking (BP) and pallet picking (PP) (see Constraint (28)).

Table 4: blackblueBorder of line usage in the current and optimal assignment

Stations	AS	US		LS		T1S		T2S		T3S		Racks	
		CA	OA	CA	OA	CA	OA	CA	OA	CA	OA	CA	OA
1884	3	1	1	2	2	-	-	-	-	-	-	-	-
2195	7	3	-	2	6	-	-	-	-	1	-	1	1
2196	5	-	-	2	3	1	-	-	-	-	-	2	2
2197	7	4	-	1	4	-	-	-	-	-	-	2	3
2198	7	2	-	3	6	-	-	1	-	-	-	1	1
2199	2	-	-	-	1	-	-	-	-	1	-	1	1
2200	3	-	-	-	2	-	-	-	-	1	-	2	1

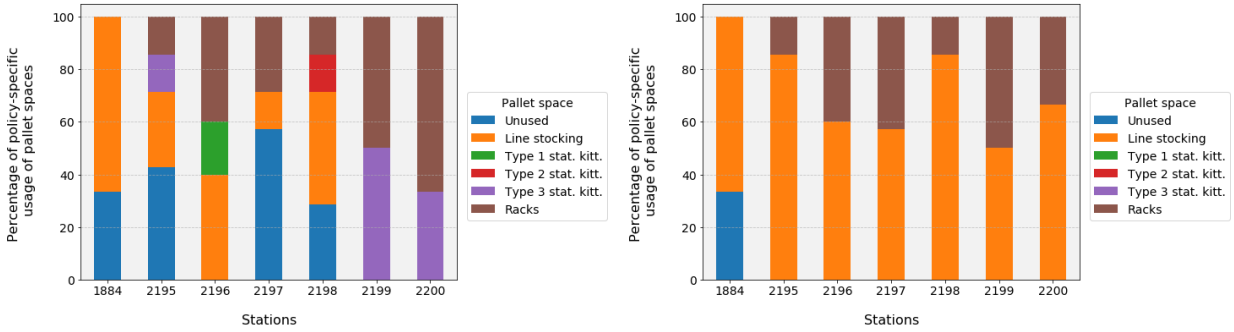
CA: Current assignment, OA: Optimal assignment, LS: Line stocking
AS: Available space, US: Unused space, T1S: Type 1 stationary kitting
T2S: Type 2 stationary kitting, T3S: Type 3 stationary kitting

According to Table 4, every station except station 1884 has no unused space in the optimal assignment. This result is unsurprising as the model maximizes the available space at

Table 5: Rack usage in the current and optimal assignment

Stations	BSS		BSM		BSL		BP		PP	
	CA	OA	CA	OA	CA	OA	CA	OA	CA	OA
1884	-	-	-	-	-	-	-	-	-	-
2195	8	-	3	5	-	4	-	1	-	-
2196	25	15	4	11	2	5	-	-	1	-
2197	27	3	6	16	-	12	-	-	1	-
2198	11	3	1	4	-	4	-	-	-	-
2199	13	3	1	9	-	2	2	1	-	-
2200	22	21	3	2	5	-	-	2	2	-

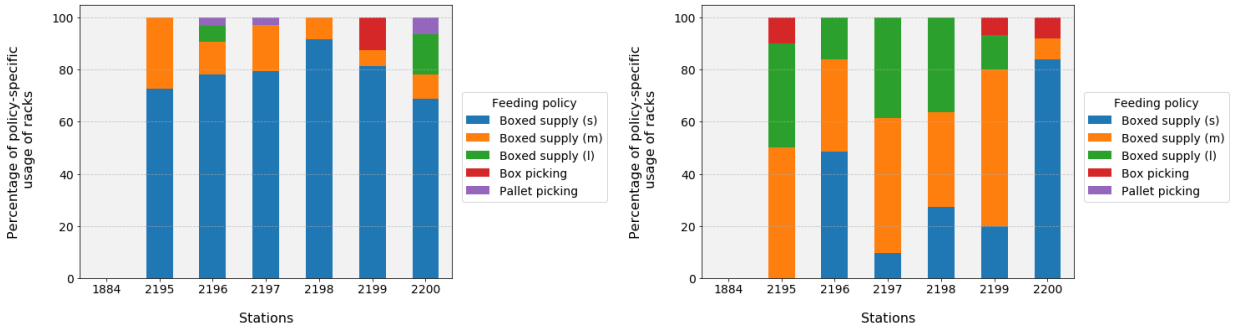
CA: Current assignment, OA: Optimal assignment
BSS: Boxed supply (small), BSM: Boxed supply (medium)
BSL: Boxed supply (large), BP: Box picking
PP: Pallet picking



(a) Current assignment

(b) Optimal assignment

Figure 4: Space usage at the blackblueborder of line



(a) Current assignment

(b) Optimal assignment

Figure 5: Rack usage at the blackblueborder of line

the blackblueborder of line as much as possible. By contrast, up to four stations have some unused spaces in the current assignment. To be fair in comparison and quantify the effect of the space usage in the current assignment, we restrict the available space at the blackblueborder of line in the optimal assignment to the space used in the current assignment. We show the result with other exciting cases in the multi-case analysis section.

6 Multi-case Analysis

This section aims to gain further insights by comparing the current situation and optimal solution with multiple settings required by the company. These settings are either borne

out of some observations following the optimal assignment or the company's specific request. Table 6 depicts the settings alongside the motivation.

Table 6: Test settings

Base settings	**Motivation**
No line stocking	Disallow the usage of line stocking in order to quantify its impact on the current situation and the optimal solution.
No boxed supply (small)	Disallow the usage of boxed supply (small) in order to quantify its impact on the current situation and the optimal solution.
No boxed supply (medium)	Disallow the usage of boxed supply (medium) in order to quantify its impact on the current situation and the optimal solution.
No boxed supply (large)	Disallow the usage of boxed supply (large) in order to quantify its impact on the current situation and the optimal solution.
No box picking	Disallow the usage of box picking in order to quantify its impact on the current situation and the optimal solution.
No traveling kitting	Disallow the usage of type 1 and type 2 traveling kitting in order to quantify its impact on the current situation and the optimal solution.
No forklift	Three parts are assigned to forklift in the optimal solution. Disallowing forklift usage will give room to quantify its impact on the current situation and the optimal solution.
No acquisition cost	To quantify the impact of vehicle type's acquisition cost on the current assignment and the optimal solution, it is imperative to set the acquisition cost to zero.
Increased distance	By increasing the distance between warehouses and the assembly stations, we aim to quantify its impact on the current assignment and the optimal solution.

While the first seven settings concern tweaking some of the decision variables in the model, the last two center on adjusting some parameters. As discussed in the space usage at the blackblueborder of line section, each setting splits into two cases, namely: (i) when the available space at the blackblueborder of line is unaltered and (ii) when the available space at the blackblueborder of line is the same as the blackblueborder-of-line space usage in the current assignment. Consequently, this analysis includes 18 cases (see Tables 7 and 8). In the first twelve cases, part assignment to the feeding policies that appear in the optimal solution is not allowed to assess the importance of these feeding policies. The feeding policies are line stocking, boxed supply (small, medium, large), box picking, and (type 1 and type 2) traveling kitting. For instance, in case 1 (*No line stocking*) and case 2 (*No line stocking**), part assignment to line stocking is disallowed. While available space at the blackblueborder of line is unaltered in case 1, the available space at the blackblueborder of line is the same as the current assignment's blackblueborder-of-line space usage in case 2. In case 13 and case 14, parts are restricted from being assigned to forklifts as only three parts get forklift assignments in the optimal assignment when the available blackblueborder-of-line space is unaltered. Furthermore, the goal of case 15 and case 16 is to allow part assignment to feeding policies and vehicle types independently of the vehicle types' acquisition cost. Case 17 and case 18 represent a situation where the warehouses, preparation areas, and supermarket are further away from the assembly stations. Thus, it will be possible to assess the effect of increased distances between these locations on the current assignment and the optimal solution.

Table 7 shows the overview of costs across the cases. Moreover, Table 8 depicts the distribution of parts to feeding policies and vehicle types across the cases. Figure 6, the graphical representation of Table 8, depicts the percentage of part assignment to feeding policies and vehicle types across the cases considered. Across all cases, as expected, we observe that space at the blackblueborder of line is of utmost importance as it translates to cost savings. The cases whose available space at the blackblueborder of line is restricted to

Table 7: Overview of costs across multiple cases

SN	Cases	Cost(€)						%ΔCA	%ΔOA
		Total	**Rep.**	**Pre.**	**Tra.**	**Usa.**	**Acq.**		
	Current assignment	4471	53	695	523	1291	1908	-	
	Optimal assignment	1962	-	204	88	1284	385	-56%	
	Optimal assignment*	2053	-	295	100	1273	385	-54%	
1	No line stocking	2792	-	1102	245	1237	209	-38%	42%
2	No line stocking*	2843	-	1152	247	1236	209	-36%	38%
3	No boxed supply (s)	2037	-	245	110	1297	385	-54%	4%
4	No boxed supply (s)*	2166	-	420	98	1263	385	-52%	6%
5	No boxed supply (m)	1979	-	219	91	1284	385	-56%	1%
6	No boxed supply (m)*	2075	-	313	104	1273	385	-54%	1%
7	No boxed supply (l)	1979	-	224	90	1284	382	-56%	1%
8	No boxed supply (l)*	2120	-	359	116	1263	382	-53%	3%
9	No box picking	1991	-	221	102	1284	385	-55%	1%
10	No box picking*	2082	-	312	113	1273	385	-53%	1%
11	No traveling kitting	2179	-	450	43	1302	385	-51%	11%
12	No traveling kitting*	2316	-	558	72	1302	385	-48%	13%
13	No forklift	1962	-	204	89	1284	385	-56%	0%
14	No forklift*	2053	-	295	100	1273	385	-54%	0%
15	No acquisition cost	1500	-	204	12	1284	-	-66%	-24%
16	No acquisition cost*	1585	-	295	18	1273	-	-65%	-23%
17	Increased distance	2173	-	319	172	1297	385	-51%	11%
18	Increased distance*	2291	-	297	135	1273	587	-49%	12%

*Available space at the blackblueborder of line is the same as blackblueborder-of-line space usage in the current assignment

SN: Serial number, Rep.: Replenishment, Pre.: Preparation, Tra.: Transportation
Usa.: Usage, Acq.: Acquisition, %ΔCA: Percentage change w.r.t. current assignment
%ΔOA: Percentage change w.r.t. optimal assignment, s: small, m: medium, l: large

the blackblueborder-of-line space usage in the current assignment have higher total costs when compared with the corresponding cases with unaltered blackblueborder-of-line space usage. Furthermore, part assignment to line stocking is higher in cases with unaltered blackblueborder-of-line available space than those with restricted blackblueborder-of-line space.

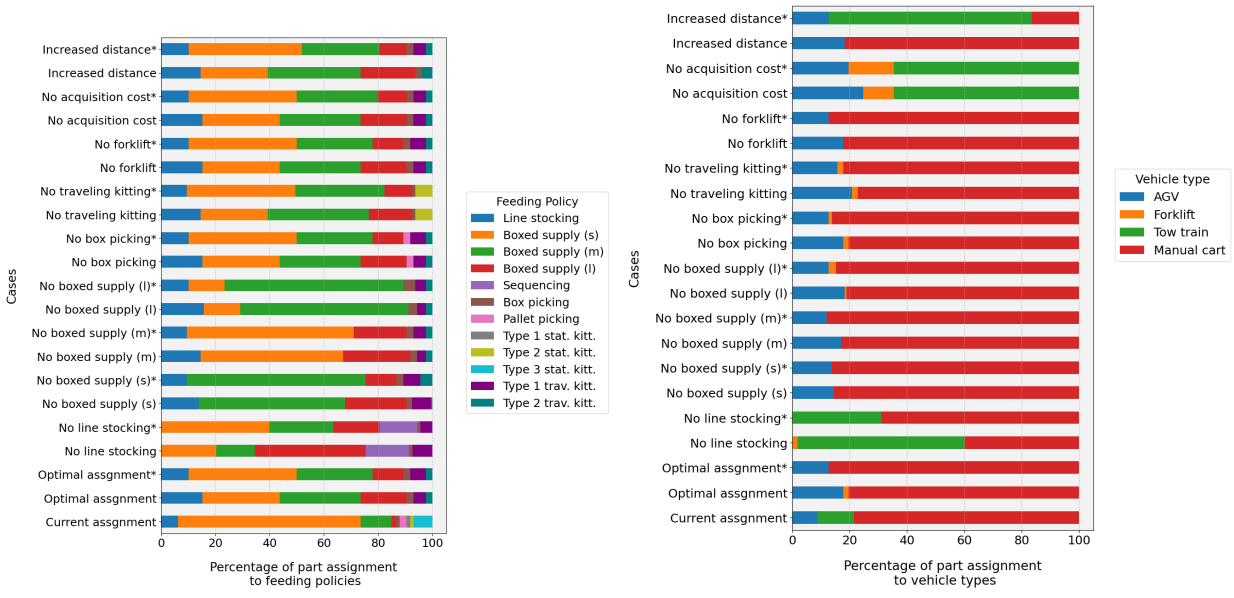
In case 1 through case 12, on the one hand, the importance of every feeding policy that appears in the optimal solution is shown through the part assignment and total cost of each case. Of all the feeding policies, line stocking is the most significant as the absence of line stocking forces the model to assign parts to sequencing, further increasing the feeding costs due to the preparation process attached to sequencing. As shown in Table 7, when line stocking is unused, up to an additional 42% and 38% of the optimal cost will have to be spent when blackblueborder-of-line available space is unaltered and restricted, respectively. Next to line stocking is traveling kitting since without the inclusion of (type 1 and type 2) traveling kitting, approximately 11% and 13% of the optimal cost will be additionally spent when blackblueborder-of-line available space is unaltered and restricted, respectively. The impact of boxed supply (small, medium, and large) also cannot be underestimated. However, the effect of line stocking and traveling kitting on the optimal solution is superior to boxed supply's. On the other hand, each of these cases outperforms the current assignment concerning cost savings (see Table 7).
The total cost realized from case 13 and case 14 is equivalent to the corresponding optimal cost. The essence of verifying the cases is to see the impact of the three parts delivered by forklift when blackblueborder-of-line available space is unaltered. According to the result, manual carts transport those parts. However, there is no effect on the total cost.

Table 8: Part distribution to feeding policies and vehicle types across multiple cases

SN	Cases	Feeding policies												Vehicle types			
		LS	BSS	BSM	BSL	SEQ	BP	PP	T1S	T2S	T3S	T1T	T2T	AGV	FL	TT	MC
	Current assignment	10	106	18	3	-	2	4	2	2	11	-	-	14	-	20	124
	Optimal assignment	24	45	47	27	-	4	-	-	-	-	7	4	28	3	-	127
	Optimal assignment*	16	63	44	18	-	4	-	-	-	-	9	4	20	-	-	138
1	No line stocking	-	33	23	66	26	2	-	-	-	-	12	-	-	3	92	63
2	No line stocking*	-	63	37	27	22	2	-	-	-	-	7	-	-	-	49	109
3	No boxed supply (s)	22	-	85	36	-	3	-	-	-	-	11	1	23	-	-	135
4	No boxed supply (s)*	15	-	104	18	-	4	-	-	-	-	10	7	22	-	-	136
5	No boxed supply (m)	23	83	-	39	-	4	-	-	-	-	5	4	27	-	-	131
6	No boxed supply (m)*	15	97	-	31	-	4	-	-	-	-	7	4	19	-	-	139
7	No boxed supply (l)	25	21	98	-	-	5	-	-	-	-	5	4	29	1	-	128
8	No boxed supply (l)*	16	21	104	-	-	7	-	-	-	-	6	4	20	4	-	134
9	No box picking	24	45	47	27	-	-	4	-	-	-	7	4	28	3	-	127
10	No box picking*	16	63	44	18	-	-	4	-	-	-	9	4	20	2	-	136
11	No traveling kitting	23	39	59	25	-	2	-	-	10	-	-	-	33	3	-	122
12	No traveling kitting*	15	63	52	16	-	2	-	-	10	-	-	-	25	3	-	130
13	No forklift	24	45	47	27	-	4	-	-	-	-	7	4	28	-	-	130
14	No forklift*	16	63	44	18	-	4	-	-	-	-	9	4	20	-	-	138
15	No acquisition cost	24	45	47	27	-	4	-	-	-	-	7	4	39	17	102	-
16	No acquisition cost*	16	63	47	17	-	4	-	-	-	-	7	4	31	25	102	-
17	Increased distance	23	39	54	32	-	4	-	-	-	-	-	6	29	-	-	129
18	Increased distance*	16	66	45	16	-	4	-	-	-	-	7	4	20	-	112	26

*Available space at the blackblueborder of line is the same as blackblueborder-of-line space usage in the current assignment

SN: Serial number, LS: Line stocking, BSS: Boxed supply (small), BSM: Boxed supply (medium)
BSL: Boxed supply (large), SEQ: Sequencing, BP: Box picking, PP: Pallet picking
T1S: Type 1 stationary kitting, T2S: Type 2 stationary kitting, T3S: Type 3 stationary kitting
T1T: Type 1 traveling kitting, T2T: Type 2 traveling kitting, AGV: Automated Guided Vehicle
FL: Forklift, TT: Tow Train, MC: Manual carts, s: small, m: medium, l: large



(a) Feeding policies

(b) Vehicle types

Figure 6: Percentage of part assignment across all cases

Interestingly, when there is no acquisition cost associated with any vehicle type, tow trains deliver approximately 65% of all parts to the blackblueborder of line in case 15 and case 16. This result contrasts the optimal assignment where tow trains are unused. It is, therefore, trivial to see that the tow train's acquisition cost has mainly contributed to its disappearance in the optimal assignment. As expected, due to inconsideration

of acquisition costs, cost savings are attributed to these cases upon comparison with the current and optimal assignment. In case 17 and case 18, the distances between the warehouses/preparation areas/supermarket and the blackblueborder of line increased by 160m. As stated earlier, this assesses the impact of increased distances on the current and the optimal assignment. The results indicate that approximately 11% and 12% of the optimal cost will be spent when the available space at the blackblueborder of line remains unaltered and when it is restricted, respectively.

7 blackblueImplementation blackblueResults

blackblueImplementing the optimal solution directly to the company's shop floor was impossible due to the increased amount of transactions needed in the automated warehouses with the optimal assignment, which would require their resizing. Consequently, the company focuses on an alternative setting that circumvents the need for warehouse resizing. Table 9 depicts the setting.

Table 9: blackblueFurther test setting

Base setting	**Motivation**
Fixed warehouse	Some parts got allocated to different warehouses in the current and optimal assignment. Therefore, this solution forbids changing the part assignment to another warehouse.

blackblueThe setting in Table 9 centers on tweaking certain decision variables within the model. Reiteratingly, each setting splits into two cases, namely: (i) when the available space at the border of line is unaltered and; (ii) when the available space at the border of line is restricted to the border-of-line space usage in the current assignment (see the space usage at the blackblueborder of line section). In the current assignment, each part has a dedicated warehouse (box, pallet, or manual warehouse) where it is initially stored. However, in the optimal assignment, it is observed that some parts switched warehouses. Consequently, case 19 and case 20 represent the situation where parts do not change warehouses as well as (i) and (ii) stated above, respectively. These cases limit part assignment to feeding policies. Conversely, the constraint ensures that the capacity of each warehouse remains intact.

Table 10: blackblueOverview of costs across the fixed-warehouse cases

SN	**Cases**	**Cost(€)**						**%ΔCA**	**%ΔOA**
		Total	**Rep.**	**Pre.**	**Tra.**	**Usa.**	**Acq.**		
	Current assignment	4471	53	695	523	1291	1908	-	
	Optimal assignment	1962	-	204	88	1284	385	-56%	
	Optimal assignment*	2053	-	295	100	1273	385	-54%	
19	Fixed warehouse	3290	53	607	575	1292	763	-26%	68%
20	Fixed warehouse*	3426	53	612	705	1292	763	-23%	67%

*Available space at the border of line is the same as border-of-line space usage in the current assignment
SN: Serial number, Rep.: Replenishment, Pre.: Preparation, Tra.: Transportation
Usa.: Usage, Acq.: Acquisition, %ΔCA: Percentage change w.r.t. current assignment
%ΔOA: Percentage change w.r.t. optimal assignment

blackblueTable 10 presents the overview of costs across the two cases alongside the current and optimal solution. Additionally, Table 11 illustrates the distribution of parts to feeding policies and vehicle types across these cases. Figure 7, a graphical representation

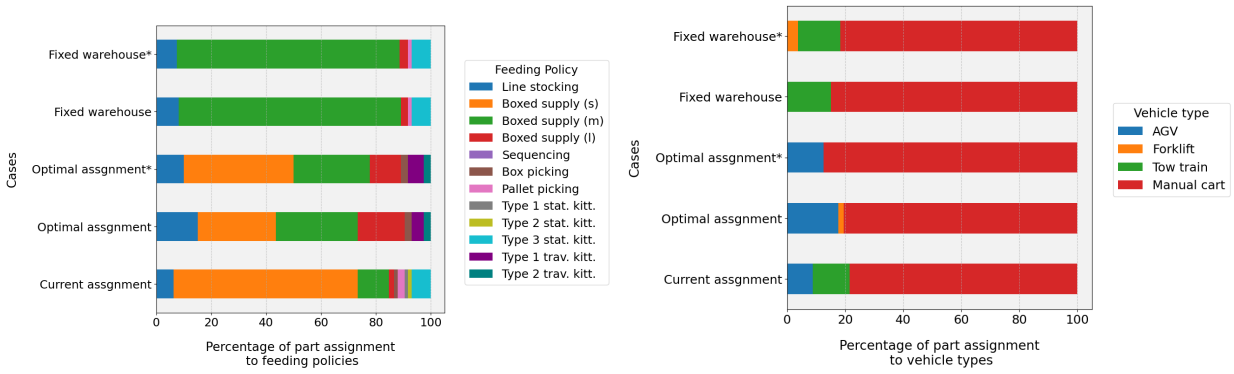
of Table 11, depicts the percentage of part assignment to feeding policies and vehicle types across these cases. The utilization of space at the blackblueborder of line plays a crucial role, resulting in cost savings across these cases. Specifically, cases constrained by limited blackblueborder-of-line space exhibit higher total costs compared to those with unrestricted blackblueborder-of-line space utilization.

Table 11: blackbluePart distribution to feeding policies and vehicle types across the fixed-warehouse cases

SN	Cases	Feeding policies												Vehicle types			
		LS	BSS	BSM	BSL	SEQ	BP	PP	T1S	T2S	T3S	T1T	T2T	AGV	FL	TT	MC
	Current assignment	10	106	18	3	-	2	4	2	2	11	-	-	14	-	20	124
	Optimal assignment	24	45	47	27	-	4	-	-	-	-	7	4	28	3	-	127
	Optimal assignment*	16	63	44	18	-	4	-	-	-	-	9	4	20	-	-	138
19	Fixed warehouse	13	-	128	4	-	-	2	-	-	11	-	-	-	-	24	134
20	Fixed warehouse*	12	-	128	5	-	-	2	-	-	11	-	-	-	6	23	129

*Available space at the border of line is the same as border-of-line space usage in the current assignment
SN: Serial number, LS: Line stocking, BSS: Boxed supply (small), BSM: Boxed supply (medium)
BSL: Boxed supply (large), SEQ: Sequencing, BP: Box picking, PP: Pallet picking
T1S: Type 1 stationary kitting, T2S: Type 2 stationary kitting, T3S: Type 3 stationary kitting
T1T: Type 1 traveling kitting, T2T: Type 2 traveling kitting, AGV: Automated Guided Vehicle
FL: Forklift, TT: Tow Train, MC: Manual carts

blackblueIn case 19 and case 20, it is interesting to see that, although parts are fixed to their dedicated warehouses as given in the current assignment, blackbluesignificant cost savings are realized when comparing these cases with the current assignment. Approximately 26% and 23% of the current assignment's cost get saved when blackblueborder-of-line available space is unaltered and restricted, respectively. Furthermore, comparing both cases to the current assignment, the number of feeding policies has been drastically reduced. Remarkably, boxed supply (small) is rendered obsolete as the waiting time during preparation attached to the feeding policy is considered to be high (see the cost component analysis section). Next to the benefit of decreasing processing costs, this reduction in number of feeding policies fosters the streamlining of the feeding process leading to smoother and less complex intralogistics flows.



(a) Feeding policies

(b) Vehicle types

Figure 7: Percentage of part assignment across the fixed-warehouse cases

blackblueThe company is now rolling out the results outside the preassembly area to the main assembly lines and the entirety of the plant. The process simplifications, due to the reduction in the number of different line feeding flows, are also enabling them to further explore opportunities for material handling process automation.

8 Conclusion and Recommendation

This paper validates a mixed integer linear programming model that minimizes total feeding and acquisition costs by integrating assembly line feeding and vehicle type decisions in a machining company. The paper contributes to the literature in several ways, namely: (*i*) for the first time, a real-life case study validates an optimization model that integrates vehicle type and assembly line feeding decisions for every part; (*ii*) to get the best out of the line feeding decisions, the model included 12 feeding policies, i.e., line stocking, boxed supply (small, medium, large), sequencing, box picking, pallet picking, (type 1, type 2, type 3) stationary kitting, (type 1 and type 2) traveling kitting; (*iii*) we explore blackblueborder-of-line space usage and show that more available space at the blackblueborder of line may affect the total feeding costs positively, and vice versa.
According to the results, the optimal assignment demonstrates superior cost savings compared to the company's current assignment, accounting for 56% of the total costs associated with the current assignment. Since the company could not immediately implement the optimal solution due to the resizing requirement of the warehouses, case 19 is proposed, not requiring warehouse resizing but still entailing a decrease of total costs of more than 20% and an improved material flow in terms of more uniform material delivery.

In this case study, part demand is deterministic, i.e., there is certainty with part demand. Future work may build on this and consider stochastic part demand. When doing so, considering stock-out risks in conjunction with delivery frequencies could prove fruitful. The number of facings required for part storage at the blackblueborder of line is fixed in this research, i.e., for every part, the model foresees the number of facings needed to store those parts during the planning horizon. However, considering variations in the number of facings required for every part due to varying part demand across different times in the planning horizon could lead to better blackblueborder-of-line space management and, ultimately, additional cost minimization. Specifically, future research may build upon this work and include a time-indexed approach, incorporating more flexible use of the blackblueborder of line over the planning horizon. Incorporating other decisions into the model, such as the size and location of preparation areas or supermarkets, could lead to exciting discoveries further.

Appendices

Mathematical model

We present an extension of the model described by Adenipekun et al. (2022) with adjusted cost calculations below. Every part is assigned to a single feeding policy p and a single vehicle type m in the model. Since this research is on a case study, the feeding policies considered are adapted to match those of the company. The feeding policies are line stocking ($p = 1$), boxed supply (small) ($p = 2$), boxed supply (medium) ($p = 3$), boxed supply (large) ($p = 4$), sequencing ($p = 5$), box picking ($p = 6$), pallet picking ($p = 7$), type 1 stationary kitting ($p = 8$), type 2 stationary kitting ($p = 9$), type 3 stationary kitting ($p = 10$), type 1 traveling kitting ($p = 11$), and type 2 traveling kitting ($p = 12$). The four vehicle types are AGVs ($m = A$), forklifts ($m = F$), tow trains ($m = T$), and manual carts ($m = C$). Furthermore, we identified six different material flows in this case study. Transportation flows describe the policy-specific movement of parts between various locations on the shop floor. These flows are enumerated as follows: the flow between the pallet warehouse and the blackblueborder of line ($w = 1$), the flow between the pallet preparation area and the blackblueborder of line ($w = 2$), the flow between box preparation area and the blackblueborder of line ($w = 3$), the flow between supermarket and the blackblueborder of line ($w = 4$), the flow between pallet preparation area and the logistical zone ($w = 5$), and the flow between box preparation area and the logistical zone ($w = 6$). Finally, some features, referred to as part attributes, are associated with each part. The part attributes are volume ($a = v$) and weight ($a = w$). The notations used in the model formulation are given in Tables 12-14. blackblueWe remark that Table 12 should be read by rows.

Table 12: Definition of sets used in the model

Sets			
$\mathcal{A}$	Set of part attributes (indexed a)	$\mathcal{E}$	Set of end products (indexed e)
$\mathcal{E}_i$	Set of end products that use part i (indexed e)	$\mathcal{F}$	Set of part families (indexed f)
$\mathcal{F}_s$	Set of part families at station s (indexed f)	$\mathcal{I}$	Set of parts (indexed i)
$\mathcal{I}_f^1$	Set of parts in family f (indexed i)	$\mathcal{I}_s$	Set of parts at station s (indexed i)
blackblue$\mathcal{I}'$	blackblueSet of parts comprising a random blackbluesingle part from each subset $\mathcal{I}_f^1$ blackblue(indexed i)	blackblue$\mathcal{I}'_s$	blackblueSet of parts comprising a random blackblue single part from each subset $\mathcal{I}_f^1$ blackblueat station s (indexed i)
$\mathcal{M}$	Set of vehicle types (indexed m)	$\mathcal{M}'$	Set of vehicle types excluding tow train (indexed m)
$\mathcal{M}_p$	Set of vehicle type m usable for feeding policy p (indexed m)	$\mathcal{P}$	Set of line feeding policies (indexed p)
$\mathcal{P}_i$	Set of line feeding policies usable for part i (indexed p)	$\mathcal{P}_w^1$	Set of line feeding policies that use transportation flow w (indexed p)
$\mathcal{R}$	Set of routes (indexed r)	$\mathcal{R}_s$	Set of routes covering assembly station s (indexed r)
$\mathcal{S}$	Set of assembly stations (indexed s)	$\mathcal{S}_r$	Set of assembly stations served by route r (indexed s)
$\mathcal{T}$	Set of takts (indexed t)	$\mathcal{V}_i$	Set of variants of part i in the family of part i (indexed i)
$\mathcal{W}$	Set of transportation flows (indexed w)	$\mathcal{W}_p$	Set of transportation flows for feeding

policy p (indexed w)

Table 13: Definition of parameters used in the model

Parameters	Meaning	Value
$blackbluea'$	Incremental aisle length in supermarket corresponding to parts assigned to type 3 stationary kitting	$1[m]$
b^s	Time needed to move kits between box warehouse and box preparation area exit	0.0433[hr]
b^{sw}	Time needed to move kits between box preparation area and box warehouse	0.0211[hr]
blackbluec^a_m	Acquisition and maintenance cost for vehicle type m per takt	$\{A: 1.04, F: 0.97, T: 0.56, C: 0.01\}[\frac{€}{\text{takt}}]$
c^c	Conveyor cost incurred at the end of the preparation process	$0\ [\frac{€}{\text{hr}}]$
blackbluec^o_m	Cost of moving vehicle type m on the shop floor	$\{A: 0, F: 25, T: 25, C: 25\}[\frac{€}{\text{hr}}]$
c^a	Wage of an assembly operator	$30[\frac{€}{\text{hr}}]$
c^l	Wage of a logistical operator	$25[\frac{€}{\text{hr}}]$
$blackbluec^1_{f_im}$	Transportation cost for family of part i provided by vehicle type m to the blackblueborder of line	blackblueEquation (49)
$blackbluec^2_{pme}$	Transportation cost of a type 1 type 2 stationary kit, type 1, or type 2 traveling kit provided by vehicle type $m \in \mathcal{M}'$ for end product e to the blackblueborder of line	blackblueEquations (52) - (59)
$blackbluec^3_{pm}$	Transportation cost of box/pallet picking for parts provided in pallets by vehicle type m to the logistical zone	blackblueEquation (50)
$blackbluec^4_{ipm}$	Costs of providing part i with feeding policy p and vehicle type m to the blackblueborder of line	blackblueEquation (40)
$blackbluec^5_{trw}$	Milk run transportation cost for deliveries made with a tow train when transporting parts in flow w over route r with frequency t	blackblueEquation (60)
d	Number of days in the horizon	20
d_e	Demand for product e	$[1-41]$
d^{ms}	Two-way distance between manual warehouse and supermarket	$2 \cdot 10$[m]
d^{bs}_s	Two-way distance between box preparation area and station s	$[77-116.3]$[m]
d^1_p	Depth of rack used for feeding policy p (number of containers)	$\{n.a.; 3; 3; 2; n.a.; 3; 3; n.a.; n.a.; n.a.; n.a.; n.a.\}$
d^{ps}_s	Two-way distance between pallet preparation area and station s	$[157-196.3]$[m]
d^{pw}_s	Two-way distance between pallet warehouse and station s	$[177-216.3]$[m]
d^l_p	Two-way distance between box/pallet	$[2 \cdot 30,\ 2 \cdot 70]$[m]

	picking preparation area and the logistical zone	
d_s^{1l}	Two-way distance between logistical zone and station s	$[77 - 116.3]$[m]
f_i	Family of part i	$[1 - 149]$
H	Planning horizon	360 [takts]
blackblueh_{ip}	Preparation handling time required for part i corresponding to line feeding policy $p \in P'$	$[3.17 \cdot 10^{-8} - 0.021]$[hr]
h_p^1	Number of shelves in a rack used for feeding policy p containers (number of containers)	$\{n.a.; 3; 3; 3; n.a.; 3; 3; n.a.; n.a.; n.a.; n.a.; n.a.\}$
l_{pA}	AGV loading and unloading time for feeding policy p	$\{0.00694; n.a.; n.a.; n.a.; 0.00694; 0.00694; 0.00694; 0.00174; 0.00694, n.a.; 0.00174; 0.00174\}$[hr]
l_{pF}	Forklift loading and unloading time for feeding policy p	$\{0.00347; 0.0000868; 0.0000723; 0.000217; 0.00347; 0.000139; 0.00347; 0.000868; 0.00347, n.a.; 0.000868; 0.000868\}$[hr]
l_{pT}	Tow train loading and unloading time for feeding policy p	$\{0.000926; 0.0000231; 0.0000193; 0.000174; 0.000926; 0.0000309; 0.000926; 0.000231; 0.000926, 0.003571; 0.000231; 0.000231\}$[hr]
l_{pC}	Manual cart loading and unloading time for feeding policy p	$\{0.00347; 0.0000868; 0.0000723; 0.000217; 0.00347; 0.000139; 0.00347; 0.000868; 0.00347, n.a.; 0.000868; 0.000868\}$[hr]
L_s	blackblueBorder-of-line capacity at station s (pallet space)	$[2 - 7]$
m_{ie}	Number of units of part i needed for product e	$[1 - 30]$
blackbluem_{r1}^r	Milk run length for a route from the pallet warehouse	213.44[m]
blackbluem_{r2}^r	Milk run length for a route from the pallet preparation area	194.2[m]
blackbluem_{r3}^r	Milk run length for a route from the box preparation area	114.2[m]
blackbluem_{r4}^r	Milk run length for a route from the supermarket	236.06[m]
$n_{f_i}^1$	Number of parts in the family of part i that fit into a sequenced container	$min\{n_{i1}, 20\}$
n_p^l	Number of policy $p \in \{6, 7\}$ parts that can be delivered to the blackblueborder of line by a manual distributor	$\{n.a.; n.a.; n.a.; n.a.; n.a.; 2; 2; n.a.; n.a.; n.a.; n.a.; n.a.\}$
n_{ip}	Number of parts that fit into a bin or container used for policy p	$\{2 - 1325536; 1 - 4645; 1 - 7897; 1 - 60000; n.a.; 1 - 4645; 1 - 7897; n.a.; n.a.; n.a.; n.a.; n.a.\}$
n_{pm}^2	Number of feeding policy p bins or containers that fit into a vehicle type m	$\{A : [1, n.a., n.a., n.a., 1, 1, 1, 1, 1, n.a., 1, 1]; F : [1, 56, 48, 16, 1, 1, 1, 1, 1, n.a., 1, 1]; T : [3, 168, 144, 16, 3, 3, 3, 3, 3, 7, 3, 3]; C : [1, 56, 48, 16, 1, 1, 1, 1, 1, n.a., 1, 1]\}$
n_p^k	Number of stationary kits for feeding policy $p \in \{8, 9, 10\}$ that fit into a kit container	$\{4, 1, 1\}$
n_p^t	Number of traveling kits for feeding policy $p \in \{11, 12\}$ that fit into a kit container	$\{4, 4\}$
$blackblueo^v$	Walking velocity of an operator	$3600[\frac{\text{m}}{\text{hr}}]$
p^{sw}	Time needed to move kits between pallet preparation area and pallet	0.0511[hr]

	warehouse	
p^w	Time needed to move kits between pallet warehouse and pallet warehouse exit	0.0094[hr]
p_i	Pick quantity for part i	$\frac{1}{\lvert\mathcal{E}\rvert}\sum_{e\in\mathcal{E}:m_{ie}\neq 0} m_{ie}$
q_p	Minimum number of facings for containers of feeding policy p,	$\{1;1;1;1;1;1;1;1;1;1;n.a.;n.a.\}$
r_{iv}	Volume of part i	$[6.2\cdot 10^{-7}-0.408][\mathrm{m}^3]$
r_{iw}	Weight of part i	$[1\cdot 10^{-5}-150][\mathrm{kg}]$
r_{vp}	Maximum volume possible in a container of policy p	$\{n.a.,n.a.,n.a.,n.a.,n.a.,n.a.,n.a.,0.147744,$ $0.816,0.288,0.147744,0.147744\}[\mathrm{m}^3]$
r_{wp}	Maximum weight allowed in a container of policy p	$\{n.a.,n.a.,n.a.,n.a.,n.a.,n.a.,n.a.,$ $240,1000,60,240,240\}[kg]$
s_{ip}	Search time for part i corresponding to line feeding policy p at the preparation area and blackblueborder of line	0.0003[hr]
v_m	Velocity of vehicle type m	$\{A:5000,F:6000,T:6000,C:3600\}[\frac{\mathrm{m}}{\mathrm{hr}}]$
w_p	Number of boxes fitting in a rack shelf used for boxed supply (number of containers)	$\{n.a.;9;6;2;n.a.;6;6;n.a.;$ $n.a.;n.a.;n.a.;n.a.\}$
$blackbluew^d_p$	Walking distance at the assembly line to pick parts corresponding to feeding policy $p\in\mathcal{P}/\{11,12\}$	2.5[m]
$blackbluew^t_p$	Waiting time at preparation points to prepare policy p containers	$\{n.a.;0.00417;0.00417;0.0125;0.0125;$ $0.00417;0.0125;0.00417;0.0125,n.a.;$ $0.00417;0.0125\}$[hr]
β	Total time in one takt	0.426[hr]
λ_i	Demand for part i	$[1-10797]$
$\lambda^1_{f_i}$	Demand for family of part i	$[1-10797]$
μ_m	utilization rate for vehicle type m (based on break time for workers, idle time, and time taken for maintenance of vehicle type m)	$\{A:0.95,F:0.99,T:0.99,C:0.85\}$
μ^o	utilization rate for manual distributor (based on break time and idle time for the distributor	0.85

Objective function

Minimize:

$$\sum_{i\in\mathcal{I}}\sum_{p\in\mathcal{P}_i}\sum_{m\in\mathcal{M}_p} blackbluec^4_{ipm}x_{ipm}+\sum_{t\in\mathcal{T}}\sum_{r\in\mathcal{R}}\sum_{w\in\mathcal{W}} blackbluec^5_{trw}v_{trw}+\sum_{i\in\mathcal{I}'}\sum_{m\in\mathcal{M}_5} blackbluec^1_{f_im}x_{i5m}+$$
$$\sum_{p\in\{6,7\}}\sum_{m\in\mathcal{M}_p} blackbluec^3_{pm}p_{pm}+\sum_{m\in\mathcal{M}}\sum_{e\in\mathcal{E}}\left(\sum_{p\in\{8,9\}}\sum_{s\in\mathcal{S}} blackbluec^2_{pme}k_{pms}+\sum_{p\in\{11,12\}} blackbluec^2_{pme}t_{pm}\right)+$$
$$Hblackbluec^a_F u^{ms}_F+H\sum_{m\in\mathcal{M}}\sum_{w\in\mathcal{W}} blackbluec^a_m blackblueu^1_{mw} \quad (1)$$

The objective function (1) minimizes the following costs: (i) cost of feeding part i with feeding policy p by transportation vehicle m denoted by $blackbluec^4_{ipm}$. This cost,

Table 14: Definition of variables used in the model

Variables	
k_{pms}	$= \begin{cases} 1, \text{ if stationary kit } p \in \{8,9,10\} \text{ is conveyed by vehicle type } m \text{ to station } s \\ 0, \text{ otherwise} \end{cases}$
t_{pm}	$= \begin{cases} 1, \text{ if traveling kit } p \in \{11,12\} \text{ is conveyed by vehicle type } m \text{ to the line} \\ 0, \text{ otherwise} \end{cases}$
v_{trw}	$= \begin{cases} 1, \text{ if a tow train transporting parts in flow } w \text{ over route } r \text{ is operating with takt } t \\ 0, \text{ otherwise} \end{cases}$
x_{ipm}	$= \begin{cases} 1, \text{ if part } i \text{ is assigned to feeding policy } p \text{ and vehicle type } m \\ 0, \text{ otherwise} \end{cases}$
$blackbluef^1_{f_i s}$	Number of facings for part family f assigned to sequencing at station s
$blackbluef^2_{ips}$	Number of facings for part i assigned to feeding policy $p \in \{1,2\}$ at station s
$blackbluef^3_{ps}$	Number of facings for feeding policy p parts used as stationary kit at station s
p_{pm}	Number of box/pallet picking container (pallets) delivered to the logistical zone by vehicle type m for a preset number of end products
r_s	Number of racks in station s
$blackblueu^1_{mw}$	Number of vehicles for each type m used in transportation flow w
$blackblueu^2_{rw}$	Number of tow trains for transporting parts in flow w over route r
u_F^{ms}	Number of forklifts used between manual warehouse and supermarket

$blackbluec^4_{ipm}$, covers all costs stemming from the replenishment, preparation, transportation and usage process. This transportation cost specifically concerns parts assigned to line stocking and boxed supply (small, medium, and large) delivered by AGV, forklift, or manual carts to the assembly stations. In addition, the transportation cost also comprises cost of manually distributing box picking and pallet picking parts from the logistical zone to the assembly stations; (ii) cost of transporting parts assigned to tow trains through flow w denoted by $blackbluec^5_{trw}$; (iii) cost of transporting sequenced parts to the line by a forklift, an AGV, or a manual cart $blackbluec^1_{f_i m}$; cost of transporting parts assigned to (iv) box and pallet picking $blackbluec^3_{pm}$, (v) type 1, type 2, type 3 stationary kitting, type 1 and type 2 traveling kitting $blackbluec^2_{pme}$ through a forklift, an AGV, or a manual cart; (vi) cost of acquiring forklifts used during the replenishment process ca_F; (vii) cost of acquiring every transportation vehicle used during the transportation process blackbluec^a_m.

General constraints

$$\sum_{p \in \mathcal{P}_i} \sum_{m \in \mathcal{M}_p} x_{ipm} = 1 \qquad \forall i \in \mathcal{I} \tag{2}$$

$$\frac{d}{Hr_{a1}} \sum_{i \in \mathcal{I}} \lambda_i r_{ia} x_{ipm} \leq p_{pm} \qquad \forall p \in \{6,7\}, m \in \mathcal{M}_p, a \in \mathcal{A} \tag{3}$$

$$\frac{n_p^k}{r_{ap}} \sum_{i \in \mathcal{I}_s} \frac{r_{ia} m_{ie}}{|\mathcal{V}_i|} x_{ipm} \leq k_{pms} \qquad \forall p \in \{8,9,10\}, m \in \mathcal{M}_p, s \in \mathcal{S}, e \in \mathcal{E}, a \in \mathcal{A} \tag{4}$$

$$\frac{n_p^t}{r_{ap}} \sum_{i \in \mathcal{I}} \frac{r_{ia} m_{ie}}{|\mathcal{V}_i|} x_{ipm} \leq t_{pm} \qquad \forall p \in \{11,12\}, m \in \mathcal{M}_p, e \in \mathcal{E}, a \in \mathcal{A} \tag{5}$$

$$\sum_{m \in \mathcal{M}_p} k_{pms} \leq 1 \qquad \forall p \in \{8,9,10\}, s \in \mathcal{S} \tag{6}$$

$$\sum_{m \in \mathcal{M}_p} t_{pm} \leq 1 \qquad \forall p \in \{11, 12\} \tag{7}$$

$$x_{ipm} \in \{0, 1\} \qquad \forall i \in \mathcal{I}, p \in \mathcal{P}_i, m \in \mathcal{M}_p \tag{8}$$

$$v_{trw} \in \{0, 1\} \qquad \forall t \in \mathcal{T}, r \in \mathcal{R}, w \in \mathcal{W} \tag{9}$$

$$k_{pms} \in \{0, 1\} \qquad \forall p \in \{8, 9, 10\}, m \in \mathcal{M}_p, s \in \mathcal{S} \tag{10}$$

$$t_{pm} \in \{0, 1\} \qquad \forall p \in \{11, 12\}, m \in \mathcal{M}_p \tag{11}$$

$$p_{pm} \in \mathbb{N}_0 \qquad \forall p \in \{6, 7\}, m \in \mathcal{M}_p \tag{12}$$

$$blackblueu^2_{rw} \in \mathbb{R}^+ \qquad \forall r \in \mathcal{R}, w \in \mathcal{W} \tag{13}$$

$$u^{ms}_m \in \mathbb{N}_0 \qquad \forall m \in \mathcal{M} \tag{14}$$

$$blackblueu^1_{mw} \in \mathbb{N}_0 \qquad \forall m \in \mathcal{M}, w \in \mathcal{W} \tag{15}$$

$$blackbluef^2_{ips} \in \mathbb{N}_0 \qquad \forall i \in \mathcal{I}, p \in \{1, 2, 3, 4, 6, 7\}, s \in \mathcal{S} \tag{16}$$

$$blackbluef^1_{f_i s} blackblue \in \mathbb{N}_0 \qquad blackblue \forall s \in \mathcal{S}, i \in \mathcal{I}'_s \tag{17}$$

$$blackbluef^3_{ps} \in \mathbb{N}_0 \qquad \forall p \in \{8, 9, 10\}, s \in \mathcal{S} \tag{18}$$

$$r_s \in \mathbb{N}_0 \qquad \forall s \in \mathcal{S} \tag{19}$$

Constraint (2) ensures the assignment of each part to a feeding policy and a vehicle type suitable for the feeding process. Constraint (3) determines the number of pallets used for box-picked and pallet-picked parts. Constraint (4) determines, for each station, whether type 1, type 2, and type 3 stationary kitting are used for an end product. To this end, it is ensured for each station that the resource requirements, e.g., volume and weight of all assigned parts, do not exceed the resource capacities in a kit. Constraint (5) determines the utilization of type 1 and type 2 traveling kitting for an end product on the assembly line. In like manner, the constraint ensures that the resource requirements, e.g., volume and weight of the assigned parts, do not exceed the resource capacities in a kit. Constraint (6) ensures the usage of, at most, one type 1, type 2, and type 3 stationary kitting per product per station. Constraint (7) ensures the usage of, at most, one type 1 and type 2 traveling kitting per product. Constraints (8) to (19) define the domains of all decision variables used in the model.

Route construction

$$\sum_{t \in \mathcal{T}} \sum_{r \in \mathcal{R}_s} v_{trw} \leq 1 \qquad \forall s \in \mathcal{S}, w \in \mathcal{W}/\{5, 6\} \tag{20}$$

$$\sum_{i \in \mathcal{I}_s} \sum_{p \in \mathcal{P}^1_w} x_{ipT} \leq M \sum_{t \in \mathcal{T}} \sum_{r \in \mathcal{R}_s} v_{trw} \qquad \forall s \in \mathcal{S}, w \in \mathcal{W}/\{5, 6\} \tag{21}$$

Constraint (20) ensures the delivery of all parts made by tow train to a specific assembly station are in a single chosen route r with a selected frequency for each flow w under consideration. Constraint (21) guarantees the selection of an appropriate route and takt for a tow train, following any part assignment to the vehicle type.

Line-sided storage

$$\frac{\lambda_i t}{H n_{ip} d_p^1} x_{ipT} - M\left(1 - \sum_{r \in \mathcal{R}_s} v_{trw}\right) \leq blackblue f_{ips}^2 \qquad \forall s \in \mathcal{S}, i \in \mathcal{I}_s, p \in \{1,2,3,4\} \cap \mathcal{P}_i, t \tag{22}$$

$$blackblue\frac{\lambda_{f_i}^1 t}{H n_{f_i}^1} x_{i5T} - M\left(1 - \sum_{r \in \mathcal{R}_s} v_{tr2}\right) blackblue \leq blackblue f_{f_i s}^1 \quad blackblue \forall s \in \mathcal{S}, i \in \mathcal{I}_s', t \in \mathcal{T} \tag{23}$$

$$\frac{t}{n_p^k} k_{pTs} - M\left(1 - \sum_{r \in \mathcal{R}_s} v_{trw}\right) \leq blackblue f_{ps}^3 \qquad \forall p \in \{8,9,10\}, s \in \mathcal{S}, t \in \mathcal{T}, w \in \mathcal{W}_p \tag{24}$$

$$q_p \sum_{m \in \mathcal{M}_p} x_{ipm} \leq blackblue f_{ips}^2 \leq M \sum_{m \in \mathcal{M}_p} x_{ipm} \qquad \forall s \in \mathcal{S}, i \in \mathcal{I}_s, p \in \{1,2,3,4,6,7\} \cap \mathcal{P}_i \tag{25}$$

$$blackblue q_5 \sum_{m \in \mathcal{M}_p} x_{i5m} \leq blackblue f_{f_i s}^1 blackblue \leq M \sum_{m \in \mathcal{M}_p} x_{i5m} \quad blackblue \forall s \in \mathcal{S}, i \in \mathcal{I}_s' \tag{26}$$

$$q_p \sum_{m \in \mathcal{M}_p} k_{pms} \leq blackblue f_{ps}^3 \leq M \sum_{m \in \mathcal{M}_p} k_{pms} \qquad \forall p \in \{8,9,10\}, s \in \mathcal{S} \tag{27}$$

$$\sum_{i \in \mathcal{I}_s} \sum_{p \in \{2,3,4,6,7\}} \frac{1}{h_p^1 w_p} blackblue f_{ips}^2 \leq r_s \qquad \forall s \in \mathcal{S} \tag{28}$$

$$\sum_{i \in \mathcal{I}_s} blackblue f_{i1s}^2 + r_s + blackblue \sum_{i \in \mathcal{I}_s'} f_{f_i s}^1 + \sum_{p \in \{8,9,10\}} blackblue f_{ps}^3 \leq L_s \quad \forall s \in \mathcal{S} \tag{29}$$

Constraints (22) and (23) ensures the reservation of a sufficient number of facings to store all containers required between any two consecutive deliveries to each station by tow trains concerning line stocking, boxed supply (small, medium, large), and sequencing. Similarly, Constraint (24) ensures this for stationary kits of type 1, type 2, and type 3. Constraints (25) to (27) ensure there is enough space reserved for every part delivered to the blackblueborder of line by vehicle type $m \in \mathcal{M}$. Additionally, these constraints ensure that a corresponding facing of a feeding policy at the blackblueborder of line can only be reserved for a part if a part assignment to that feeding policy exists. The number of racks used for boxed supply (small, medium, large), box picking, and pallet picking is determined in Constraint (28). With Constraint (29), it is guaranteed that, for each station, the sum of all facings does not exceed the station's capacity.

Fleet size determination

$$\frac{t}{H} \sum_{s \in \mathcal{S}_r} \sum_{i \in \mathcal{I}_s} \frac{\lambda_i}{n_{i1}} \left(\frac{blackbluem_{r1}^r}{n_{1T} \mu_T v_T} + l_{1T}\right) x_{i1T} + \sum_{s \in \mathcal{S}_r} \frac{t}{n_9^k} \left(\frac{blackbluem_{r1}^r}{n_{9T} \mu_T v_T} + l_{9T}\right) k_{9Ts} +$$

$$\frac{t}{n_{12}^t}\left(\frac{blackbluem_{11}^r}{n_{12T}\mu_T v_T}+l_{12T}\right)t_{12T}-M(1-v_{tr1})\leq \beta t u_{r1}$$
$$\forall t\in\mathcal{T}, r\in\mathcal{R} \quad (30)$$

$$\frac{t}{H}\sum_{s\in\mathcal{S}_r}\left(\sum_{i\in\mathcal{I}_s}\frac{\lambda_i}{n_{i4}}\left(\frac{blackbluem_{r2}^r}{n_{4T}\mu_T v_T}+l_{4T}\right)x_{i4T}+blackblue\sum_{i\in\mathcal{I}_s'}\frac{\lambda_{f_i}^1}{n_{f_i}^1}\left(\frac{blackbluem_{r2}^r}{n_{5T}\mu_T v_T}+l_{5T}\right)x_{\tilde{i}5T}\right)$$
$$-M(1-v_{tr2})\leq \beta t u_{r2} \qquad \forall t\in\mathcal{T}, r\in\mathcal{R} \quad (31)$$

$$\frac{t}{H}\sum_{s\in\mathcal{S}_r}\sum_{i\in\mathcal{I}_s}\sum_{p\in\{2,3\}\cap\mathcal{P}_i}\frac{\lambda_i}{n_{ip}}\left(\frac{blackbluem_{r3}^r}{n_{pT}\mu_T v_T}+l_{pT}\right)x_{ipT}+\sum_{s\in\mathcal{S}_r}\frac{t}{n_8^k}\left(\frac{blackbluem_{r3}^r}{n_{8T}\mu_T v_T}+l_{8T}\right)k_{8Ts}+$$
$$\frac{t}{n_{11}^t}\left(\frac{blackbluem_{13}^r}{n_{11T}\mu_T v_T}+l_{11T}\right)t_{11T}-M(1-v_{tr3})\leq \beta t u_{r3}$$
$$\forall t\in\mathcal{T}, r\in\mathcal{R} \quad (32)$$

$$\sum_{s\in\mathcal{S}_r}\frac{t}{n_{10}^k}\left(\frac{blackbluem_{r4}^r}{n_{10T}\mu_T v_T}+l_{10T}\right)k_{10Ts}-M(1-v_{tr4})\leq \beta t u_{r4} \qquad \forall t\in\mathcal{T}, r\in\mathcal{R} \quad (33)$$

$$\sum_{r\in\mathcal{R}} blackblueu_{rw}^2\leq u_{Tw} \qquad \forall w\in\mathcal{W}/\{5,6\} \quad (34)$$

$$\frac{1}{\beta H}\sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}}\frac{\lambda_i}{n_{i1}}\left(\frac{d^{ms}}{n_{1F}\mu_F v_F}+l_{1F}\right)x_{i10m}\leq u_F^{ms} \quad (35)$$

$$\frac{1}{\beta H}\sum_{s\in\mathcal{S}}\sum_{i\in\mathcal{I}_s}\frac{\lambda_i}{n_{i1}}\left(\frac{d_s^{pw}}{n_{1m}\mu_m v_m}+l_{1m}\right)x_{i1m}+$$
$$\sum_{e\in\mathcal{E}}\frac{d_e}{\beta H}\left(\sum_{s\in\mathcal{S}}\frac{1}{n_9^k}\left(\frac{d_s^{pw}}{n_{9m}\mu_m v_m}+l_{9m}\right)k_{9ms}+\frac{1}{n_{12}^t}\left(\frac{d_1^{pw}}{n_{12m}\mu_m v_m}+l_{12m}\right)t_{12m}\right)\leq u_{m1}$$
$$\forall m\in\mathcal{M}' \quad (36)$$

$$\frac{1}{\beta H}\sum_{s\in\mathcal{S}}\sum_{i\in\mathcal{I}_s}\frac{\lambda_i}{n_{i4}}\left(\frac{d_s^{ps}}{n_{4m}\mu_m v_m}+l_{4m}\right)x_{i4m}+$$
$$blackblue\sum_{s\in\mathcal{S}}\sum_{i\in\mathcal{I}_s'}\frac{\lambda_{f_i}^1}{n_{f_i}^1}\left(\frac{d_s^{ps}}{n_{5m}\mu_m v_m}+l_{5m}\right)x_{i5m}\leq u_{m2}$$
$$\forall m\in\mathcal{M}' \quad (37)$$

$$\frac{1}{\beta H}\sum_{s\in\mathcal{S}}\left(\sum_{i\in\mathcal{I}_s}\sum_{p\in\{2,3\}}\frac{\lambda_i}{n_{ip}}\left(\frac{d_s^{bs}}{n_{pm}^2\mu_m v_m}+l_{pm}\right)x_{ipm}\right)+$$
$$\sum_{e\in\mathcal{E}}\frac{d_e}{\beta H}\left(\sum_{s\in\mathcal{S}}\frac{1}{n_8^k}\left(\frac{d_s^{bs}}{n_{8m}\mu_m v_m}+l_{8m}\right)k_{8ms}+\frac{1}{n_{11}^t}\left(\frac{d_1^{bs}}{n_{11m}\mu_m v_m}+l_{11m}\right)t_{11m}\right)\leq u_{m3}$$
$$\forall m\in\mathcal{M}' \quad (38)$$

$$\frac{d}{\beta H}\left(\frac{d_p^l}{n_{1m}\mu_m v_m}+l_{1m}\right)p_{pm}\leq blackblueu_{mw}^1 \qquad \forall p\in\{6,7\}, m\in\mathcal{M}_p, w\in\mathcal{W}_p \quad (39)$$

Constraints (30) to (33) compute the number of tow trains used when considering every possible route and frequency during the planning horizon by estimating the ratio of the number of takts between two consecutive deliveries and (the number of takts in) the planning horizon. It is ensured with Constraint (34) that the number of available tow trains is enough to perform all transportation demands through the routes at any takt. Within Constraint (35), the number of forklifts needed to execute the replenishment process is determined. Constraints (36) to (38) provide the number of forklifts, AGVs, and manual carts needed to perform all transportation demands. Constraint (39) provides the number of vehicle type $m\in\mathcal{M}'$ used in performing the transportation operation for parts assigned to box picking and pallet picking during the planning horizon by considering the ratio of the number of days and the number of takts in the planning horizon.

Cost parameter calculation

We present a description of the cost associated with the model in this section. Just as in Adenipekun et al. (2022), we take four distinct cost components into consideration, namely: (i) cost incurred through part replenishment at the supermarket, c_{ipm}^R; (ii) preparation cost incurred for all feeding policies except line stocking which do not require preprocessing, c_{ipm}^P; (iii) transportation cost incurred when delivering parts to the blackblueborder of line. Transportation cost of parts when using AGVs, forklifts, or manual carts is given as $blackbluec_{ipm}^T$ for parts assigned to line stocking, boxed supply (small, medium, and large), $blackbluec_{f_im}^1$, $blackbluec_{pm}^3$, and $blackbluec_{pme}^2$ for parts assigned to sequencing, box picking, pallet picking, (type 1, type 2) stationary kitting, (type 1 and type 2) traveling kitting respectively. Elsewhere, transportation cost for parts delivered by tow trains is $blackbluec_{trw}^5$; (iv) usage cost incurred at the blackblueborder of line, c_{ipm}^U (see 40).

$$blackbluec_{ipm}^4=c_{ipm}^R+c_{ipm}^P+c_{ipm}^T+c_{ipm}^U \qquad \forall i\in\mathcal{I}, p\in\mathcal{P}, m\in\mathcal{M} \quad (40)$$

The replenishment cost section through the usage cost section fully describes the costs. Table 13 delineates the notation for the parameter values used in the model. The information received from the manufacturing company accounts for these values. Set and variable notations with their corresponding meaning are depicted in Tables 12 and 14.

Replenishment cost

$$c^R_{i10F} = co_F \left[\frac{\lambda_i}{n_{i1}} \left(\frac{d^{ms}}{n_{1F}\mu_F v_F} + l_{1F} \right) \right] \qquad \forall i \in \mathcal{I} \tag{41}$$

Equation (41) provides the replenishment cost, i.e., the cost of replenishing the supermarket with parts from the manual warehouse, the product of the operating cost for a forklift for one-time unit co_F and the replenishment times result in the replenishment cost; we determine the replenishment times by dividing a part's demand λ_i by the number of parts that fit into a pallet n_{i1}, which yields the total number of pallets; we further divide the output by the number of pallets that can be loaded into a forklift at once n_{1F}, and the forklift's utilization rate μ_F, and further multiplied by the time taken by a forklift to go back and forth from the manual warehouse to the supermarket; dividing the distance covered by a forklift to go back and forth from the manual warehouse to the supermarket d^{ms} by the forklift's velocity v_F result in this time. Furthermore, we compute the product of the total number of pallets and the corresponding loading and unloading time l_{1F}.

Preparation cost

$$c^P_{ipm} = c^l \lambda_i \left[\frac{blackbluew^t_p}{n_{ip}} + blackblueh_{ip} \right] \qquad \forall i \in \mathcal{I}, p \in \{2,3,4,5\} \cap \mathcal{P}_i, m \in \mathcal{M}_p \tag{42}$$

$$c^P_{ipm} = c^l \left[blackbluew^t_p d + \lambda_i blackblueh_{ip} \right] \qquad \forall i \in \mathcal{I}, p \in \{6,7\} \cap \mathcal{P}_i, m \in \mathcal{M}_p \tag{43}$$

$$c^P_{ipm} = c^l \left[\left(blackbluew^t_p \sum_{e \in \mathcal{E}_i} d_e \right) + \lambda_i blackblueh_{ip} \right] \qquad \forall i \in \mathcal{I}, p \in \{8,9,11,12\} \cap \mathcal{P}_i, m \in \mathcal{M}_p \tag{44}$$

$$c^P_{i10m} = c^l \left[\left(\frac{blackbluea' \sum_{e \in \mathcal{E}_i} d_e}{blackblueo^v} \right) + \lambda_i blackblueh_{ip} \right] \quad \forall i \in \mathcal{I}, m \in \mathcal{M}_{10} \tag{45}$$

The logistics operator must follow a pick list when preparing parts for any feeding policies requiring preparation. The product of the supermarket logistic operator cost and the preparation times give the preparation cost. Preparation times for parts assigned to boxed supply (small, medium, or large) or sequencing depend on the part's demand λ_i divided by number of parts that can fit into the corresponding container. We further multiply the output by the supermarket logistics operator's waiting time. In addition, the handling time blackblueh_{ip} is also considered (see Equation (42)). For parts assigned to box picking and pallet picking, preparation times comprise multiplying the number of days in the planning horizon by the logistics operator's waiting time. Moreover, we add the result to the product of part's demand λ_i and the handling time blackblueh_{ip} (see Equation (43)). In type 1 and type 2 stationary kitting, type 1 and type 2 traveling kitting, part preparation times depend on the product of the logistics operator's waiting time and the sum of all products' demand using part i. We further add this result to the product of part's demand λ_i and the handling time blackblueh_{ip} (see Equation (44)). In type 3 stationary kitting, preparation times are derived from the product of all products' demand using part i and the incremental time required to walk through the supermarket

for kitted parts, i.e., $blackbluea'$ divided by the operator's velocity $blackblueo^v$. As before, part's demand λ_i and the handling time blackblueh_{ip} are also factored in (see Equation (45)).

Transportation cost

$$c^T_{i1m} = blackbluec^o_m \left[\frac{\lambda_i}{n_{i1}} \left(\frac{d^{pw}_s}{n_{1m}\mu_m v_m} + l_{1m} \right) \right] \qquad \forall s \in \mathcal{S}, i \in \mathcal{I}_s, m \in \mathcal{M}' \tag{46}$$

$$c^T_{ipm} = blackbluec^o_m \left[\frac{\lambda_i}{n_{ip}} \left(\frac{d^{bs}_s}{n^2_{pm}\mu_m v_m} + l_{pm} \right) \right] \qquad \forall s \in \mathcal{S}, i \in \mathcal{I}_s, p \in \{2,3\} \cap \mathcal{P}_i, m \in \{F, C\} \tag{47}$$

$$c^T_{i4m} = blackbluec^o_m \left[\frac{\lambda_i}{n_{i4}} \left(\frac{d^{ps}_s}{n_{4m}\mu_m v_m} + l_{4m} \right) \right] \qquad \forall s \in \mathcal{S}, i \in \mathcal{I}_s, m \in \{F, C\} \tag{48}$$

$$blackbluec^1_{f_i m} = blackbluec^o_m \left[\frac{\lambda^1_{f_i}}{n^1_{f_i}} \left(\frac{d^{ps}_s}{n_{5m}\mu_m v_m} + l_{5m} \right) \right] \qquad \forall blackblues \in \mathcal{S}, i \in \mathcal{I}'_s, m \in \mathcal{M}' \tag{49}$$

$$blackbluec^3_{pm} = blackbluec^o_m \left[d \left(\frac{d^l_p}{n_{1m}\mu_m v_m} + l_{1m} \right) \right] \qquad \forall p \in \{6,7\}, m \in \mathcal{M}_p \tag{50}$$

$$c^T_{ipm} = c^l \left[\lambda_i \left(\frac{d^{1l}_s}{n^l_p \mu^o blackblueo^v} + blackblueh_{ip} \right) \right] \qquad \forall s \in \mathcal{S}, i \in \mathcal{I}_s, p \in \{6,7\} \cap \mathcal{P}_i, m \in \mathcal{M}_p \tag{51}$$

$$blackbluec^2_{8me} = blackbluec^o_m \left[\frac{d_e}{n^k_8} \left(\frac{d^{bs}_s}{n_{8m}\mu_m v_m} + l_{8m} + \frac{c^c}{blackbluec^o_m} \left(b^{sw} + \frac{b^s}{n_{8m}} \right) \right) \right] \qquad \forall e \in \mathcal{E}, m \in \mathcal{M} \tag{52}$$

$$blackbluec^2_{9me} = blackbluec^o_m \left[\frac{d_e}{n^k_9} \left(\frac{d^{pw}_s}{n_{9m}\mu_m v_m} + l_{9m} + \frac{c^c}{blackbluec^o_m} \left(p^{sw} + \frac{p^w}{n_{9m}} \right) \right) \right] \qquad \forall e \in \mathcal{E}, m \in \mathcal{M} \tag{53}$$

$$blackbluec^2_{8Te} = c^c \left[\frac{d_e}{n^k_8} \left(b^{sw} + \frac{b^s}{n_{8m}} \right) \right] \qquad \forall e \in \mathcal{E} \tag{54}$$

$$blackbluec^2_{9Te} = c^c \left[\frac{d_e}{n^k_9} \left(p^{sw} + \frac{p^w}{n_{9m}} \right) \right] \qquad \forall e \in \mathcal{E} \tag{55}$$

$$blackbluec^2_{11me} = blackbluec^o_m \left[\frac{d_e}{n^t_{11}} \left(\frac{d^{bs}_1}{n_{11m}\mu_m v_m} + l_{11m} + \frac{c^c}{blackbluec^o_m} \left(b^{sw} + \frac{b^s}{n_{11m}} \right) \right) \right] \qquad \forall e \in \mathcal{E}, m \in \mathcal{M} \tag{56}$$

$$blackbluec^2_{12me} = blackbluec^o_m \left[\frac{d_e}{n^t_{12}} \left(\frac{d^{pw}_1}{n_{12m}\mu_m v_m} + l_{12m} + \frac{c^c}{blackbluec^o_m} \left(p^{sw} + \frac{p^w}{n_{12m}} \right) \right) \right] \qquad \forall e \in \mathcal{E}, m \in \mathcal{M} \tag{57}$$

$$blackbluec^2_{11Te} = c^c \left[\frac{d_e}{n^t_{11}} \left(b^{sw} + \frac{b^s}{n_{11m}} \right) \right] \qquad \forall e \in \mathcal{E} \tag{58}$$

$$blackbluec^2_{12Te} = c^c \left[\frac{d_e}{n^t_{12}} \left(p^{sw} + \frac{p^w}{n_{12m}} \right) \right] \qquad \forall e \in \mathcal{E} \tag{59}$$

$$blackbluec^5_{trw} = co_T \left[\frac{Hblackbluem^r_{rw}}{t \mu_T v_T} \right] \qquad \forall t \in \mathcal{T}, r \in \mathcal{R} \tag{60}$$

Transportation cost is the product of the transportation times and the vehicle type m's operating cost for one time unit $blackbluec^o_m$; transportation times for forklifts in line stocking, boxed supply (small, medium, large), and sequencing is the division of a part's demand λ_i by the number of parts that fit into the corresponding load carrier n_{ip}, which yields the total number of load carriers needing transport; the output is divided by the number of load carriers that can be loaded into a forklift at once n_{pF}, and the forklift's utilization rate μ_F, and further multiplied by the time taken by a forklift to go back and forth from the pallet warehouse, pallet preparation area or box preparation area to the blackblueborder of line; dividing the distance covered by a forklift to go back and forth from the pallet warehouse to the blackblueborder of line d^{pw}_s (or from the pallet preparation area to the blackblueborder of line d^{ps}_s or from the box preparation area to the blackblueborder of line d^{bs}_s) by forklift's velocity v_F result in the time; furthermore, we multiply the total number of load carriers needing transport by the corresponding loading and unloading time l_{pF} (see Equations (46) to (49)).

In kitted parts, i.e., type 1 and type 2 stationary kitting, type 1 and type 2 traveling kitting, forklifts' transportation times depend on the sum of all products' demand divided by number of kits that fit into a kit container n^k_p (or n^t_p), number of kit containers transportable at once n^2_{pm}, and forklifts' utilization rate μ_F. In addition, the loading and unloading time as well as conveyor time involved in moving prepared kits to their respective warehouses and back to the loading point are all taken into consideration (see Equations (52), (53), (56), and (57)); the transportation times for AGVs and manual carts are similar to forklifts for all feeding policies discussed hitherto in this section. By contrast, transportation times for a tow train depend on the number of times a milk-run route is performed on a chosen route during the planning horizon $\frac{H}{t}$, divided by the tow train's utilization rate μ_T. The result is further multiplied by the time to perform a milk run either from the pallet warehouse, pallet preparation area, box preparation area or the supermarket $\frac{blackbluem^r_{rw}}{v_T}$. Notice that $blackbluem^r_{rw}$ is the milk run's length between the pallet warehouse, pallet preparation area, box preparation area, or the supermarket and the line, while v_T represents tow train's velocity (see Equation (60)). We also consider the loading and unloading time for parts assigned to tow trains. Concerning kitted parts delivered by a tow train to the blackblueborder of line, conveyor time involved in moving prepared kits to their respective warehouses and back to the loading point is considered (see Equations (54), (55), (58), and (59)).

Transportation times for any vehicle type, in box picking and pallet picking, can be obtained by dividing the number of days in a planning horizon by the number of pallets that can be loaded into the corresponding vehicle type n_{1m} and the vehicle's utilization

rate μ_m. We further multiply the result by the time taken to move back and forth from the pallet preparation area or box preparation area to the logistical zone, dividing the distance covered by a vehicle to go back and forth from the preparation area to the logistical zone d_p^l by the vehicle's velocity v_m result in this time. Finally, the number of days in a planning horizon is multiplied by the corresponding loading and unloading time l_{1m} (see Equation (50)). Manual distributors' transportation times delivering box picking and pallet picking containers to the blackblueborder of line is similar to forklifts' transportation times. However, the two-way distance between the logistical zone and blackblueborder of line d_s^{1l}, number of containers a manual distributor can deliver at once to the blackblueborder of line n_p^l, manual distributor's utilization rate μ^o, manual distributor's velocity $blackblueo^v$ as well as part's handling time blackblueh_{ip} are all considered (see Equation (51)).

Usage cost

$$c_{ipm}^U = c^a \frac{\lambda_i}{p_i} \left(\frac{blackbluew_p^d}{blackblueo^v} + s_{ip} \right) \qquad \forall i \in \mathcal{I}, p \in \{1,2,3,4,6,7\} \cap \mathcal{P}_i, m \in \mathcal{M}_p \tag{61}$$

$$c_{ipm}^U = c^a \frac{\lambda_i}{p_i} \frac{blackbluew_p^d}{blackblueo^v} \qquad \forall i \in \mathcal{I}, p \in \{5,8,9,10\} \cap \mathcal{P}_i, m \in \mathcal{M}_p \tag{62}$$

In line stocking, boxed supply (small, medium, large), box picking, and pallet picking, we compute usage times by dividing a part's demand λ_i by the part's pick quantity p_i. The output, which is the number of times an assembly operator will move between the assembly line and blackblueborder of line, is multiplied by the walking time, i.e., the distance to go back and forth from the assembly station to the blackblueborder of line $blackbluew_p^d$ divided by operator's walking velocity $blackblueo^v$. Finally, the number of times an assembly operator has to move between the assembly line and the blackblueborder of line is multiplied by the search time s_{ip} (see Equation (61)). The usage cost presented in Equation (62) is similar to Equation (61). However, the feeding policies considered do not need to undergo any search process as parts do not require further identification.

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