Socio-Economic Planning Sciences 46 (2012) 46–54

Contents lists available at SciVerse ScienceDirect

Socio-Economic Planning Sciences

journal homepage: www.elsevier.com/locate/seps

# Pre-positioning and dynamic delivery planning for short-term response following a natural disaster

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ARTICLE INFO

*Article history:*
Available online 4 November 2011

*Keywords:*
Pre-positioning strategies
Emergency response
Stochastic programming
Facility location
Resource allocation
Dynamic models

ABSTRACT

Natural disasters often result in large numbers of evacuees being temporarily housed in schools, churches, and other shelters. The sudden influx of people seeking shelter creates demands for emergency supplies, which must be delivered quickly. A dynamic allocation model is constructed to optimize pre-event planning for meeting short-term demands (over approximately the first 72 h) for emergency supplies under uncertainty about what demands will have to be met and where those demands will occur. The model also includes requirements for reliability in the solutions – i.e., the solution must ensure that all demands are met in scenarios comprising at least 100$\alpha$% of all outcomes. A case study application using shelter locations in North Carolina and a set of hurricane threat scenarios is used to illustrate the model and how it supports an emergency relief strategy.



## 1. Introduction

Disaster management is the sequence of operations that seek to prevent or reduce the injuries, fatalities, and damages resulting from a disaster; and to facilitate the recovery from such an event. This process is comprised of four sequential stages occurring during the life cycle of the disaster (e.g., mitigation, preparedness, response and recovery). During the mitigation stage actions are taken to prevent the onset of the disaster or moderate its effects. The preparedness stage aims at decreasing the response time by the advanced procurement and pre-positioning of needed supplies. In the response stage, the disaster mitigations plans are activated and the emergency supplies are mobilized. The final steps of emergency relief (i.e., retrieving the victims, rebuilding the infrastructure, and mitigating damages in the disaster zones) occur during the recovery phase [1]. This paper focuses on the preparedness and the response stages, specifically the pre-positioning of supplies for sheltered victims and the timely distribution of the supplies during the progression of the event.

Pre-positioning of emergency supplies is a means for increasing preparedness for natural disasters. Key decisions in pre-positioning are the locations and capacities of emergency distribution centers, as well as allocations of inventories of multiple relief commodities to those distribution locations. Extreme events such as tornados, floods, earthquakes or hurricanes create situations where large numbers of people may be directed to shelters and require food, water, medical supplies, etc. for survival. For emergency response efforts to be effective in providing assistance to disaster victims, certain vital supplies should be close at hand, but the shelter locations themselves (often schools, churches, sports arenas, etc.) may not have suitable storage facilities. Furthermore, uncertainty about where (and whether) an event will occur makes it useful to consider specifically located storage facilities that can distribute materials to several different shelter locations in quantities that match needs in a specific incident.

Pre-positioning provides numerous benefits to relief organizations [1]. First, supplies stored in a pre-positioned network reduce the response time assisting disaster victims. Second, through the advanced procurement of supplies humanitarian organizations can consider local, regional and international suppliers, and attain better prices for the items. Third, by buying in larger volumes humanitarian organizations can increase their purchasing power and take advantage of lower bulk prices.

In this paper, our concern is with short-term demands at shelter locations (in approximately the first 72 h after an evacuation is ordered, or after an event occurs). During these initial hours, evacuees are arriving at shelter locations, initial provision of needed relief supplies is critical and there is little opportunity for meeting that demand from distant suppliers. The total number of

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doi:10.1016/j.seps.2011.10.002

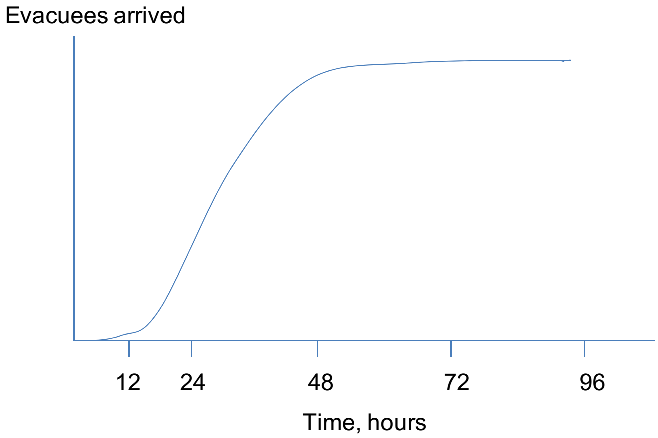


**Fig. 1.** General pattern of arrivals at a shelter location.

people responding to an evacuation order over time is frequently represented by an S-shaped curve, as shown in Fig. 1 [2,3]. The shape, height and time-scale of the curve shown in Fig. 1 will be scenario- and location-dependent. Because the number of evacuees arriving at a given shelter location is unknown prior to the event, and evolves over time during an event, a dynamic model of supply delivery is important. However, the ability of the distribution system to respond dynamically as the situation evolves is limited by pre-event planning decisions on locations and amounts of materials that are available. Thus, we want an integrated model of the pre-event planning decisions and the during-event (or immediately post-event) dynamic distribution decisions.

Several previous authors have addressed problems of post-event distribution of emergency supplies. Important examples include the work of Haghani and Oh [4], Sheu [5], Houming et al. [6] and Yan and Shi [7]. These models assume that the characteristics of the event, the resulting demands for the supplies, the locations and the available quantities of those supplies, and the condition of the transportation network are known. Haghani and Oh [4] developed a multi-commodity multi-mode network flow model to help emergency response managers organize detailed load plans for moving commodities after an event. Barbarosoglu and Arda [8] extended that model to consider uncertainties in available supplies, demands and network arc capacities through definition of a set of scenarios. This opens opportunities for connections to pre-event planning, but Barbarosoglu and Arda [8] assume that the locations and availability of supplies are given exogenously. Related work has been done by Ozdamar et al. [9], who created a dynamic model to generate multi-period vehicle routes and schedules along with commodity load-unload assignments.

A different aspect of managing uncertainty in relief supply distribution is represented in the work of Ozbay and Ozguven [10]. They propose an inventory model for determining the stocking level of a commodity subject to uncertain demands and delivery schedules at a shelter location or relief distribution point, so that there is a “stock out” probability no larger than a specified value. This inventory question is important for managing materials at the demand points, but their work does not address location issues or determination of distribution strategy over time.

Other research efforts have been aimed at devising pre-positioning plans for emergency supplies. Ukkusuri and Yushimito [11] develop a model to choose locations for pre-positioned supplies in a way that maximizes the probability that demand points can be reached from at least one supply facility under conditions where facility locations and links in the transportation network may become unusable. Akkihal [12] examines the impact of inventory pre-positioning on humanitarian operations through a supply-chain perspective. The objective of his model is to minimize the per-capita or the average global distance from the nearest warehouse to the forecasted hazard victims. Balcik and Beamon [13] develop a maximal covering location model that determines the number and location of fixed-capacity distribution centers in a relief network, and the amount of relief supplies to be stocked at each center. The model considers pre- and post-disaster budget restrictions but does not consider network reliability. Demands are specified through the use of scenarios, but there is an implicit assumption that in each scenario there is demand in only one location.

Duran et al. [14] used a mixed-integer inventory location model to evaluate the effects of pre-positioning of relief items on the average response time of a major international organization (CARE International) aiding victims of natural disasters (e.g., earthquakes, windstorms, wave surges, and floods). The solution is constrained by a maximum allowed number of warehouses and limits on inventory levels kept in the pre-positioning network. Rawls and Turnquist [15] present a two-stage stochastic programming model for determining pre-positioning storage locations and capacities, as well as allocating quantities of various emergency commodities to be stored at each location. These “first-stage” decisions (i.e., made before characteristics of any event are known) condition the potential response to specific possible events (scenarios). The responses (“second-stage” decisions) are movements of available supplies to meet demands in specific locations impacted by a given event scenario, over a transportation network that may have limited capacity due to damage incurred in the event. The two-stage model minimizes total expected costs (facility fixed charges, acquisition and storage of supplies, transportation and penalties for unmet demand) over all scenarios.

The model in Rawls and Turnquist [15] provides a springboard for the work described here. In this effort, we extend the static model of demand in each scenario from the earlier model to make it dynamic and specifically related to the arrival of evacuees at shelter locations over time. The new solution determines the most auspicious supply distribution pattern per time period and scenario based on the timely needs of the evacuees at the shelters, the limited storing capacity of the shelter, the capacity of the shipment travel modes, and the rate at which storing facilities can dispatch the supplies. Also, the new model includes the concept of reliability that guarantees with $\alpha$% certainty that demand would be met with the pre-positioned arrangement suggested by the solution.

The remainder of the paper is organized as follows. Section 2 describes the model formulation. In Section 3, we use data developed for an assessment of hurricane shelter locations in North Carolina as the basis for a case study in pre-positioning supplies. Section 4 provides conclusions and directions for further research.

## 2. Model formulation

In planning for arrivals at shelter locations, a specific policy on provision of emergency supplies forms the basis for plans. Many different policies are possible, but one example (for consumable materials) would be: “Enough material should be present at the shelter location by $t = 12$ h to handle anticipated needs over the first 48 h. By time $t = 24$ h, there should be enough material to handle the first 72 h. Each day after that, enough additional material should be received to cover one more day of anticipated needs, so that a two-day supply is maintained on hand in the event of further disruptions.” The important aspect of a policy is that it specifies the parameters that drive demands for material over time at specific shelter locations in a particular event.

The quantity implications for consumable materials (food, water, etc.) can be related to the arrival function in Fig. 1. An

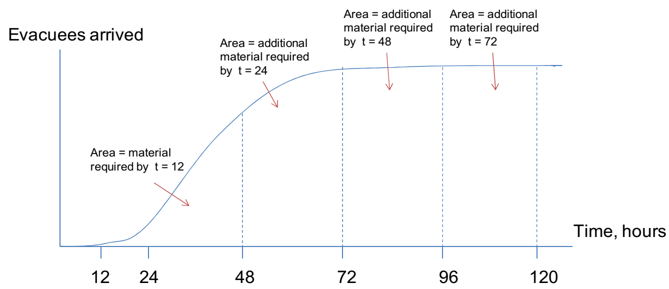


**Fig. 2.** Graphical representation of a policy on provision of consumable materials and its relation to arrival of evacuees.

example reflecting the specific policy described in the previous paragraph is illustrated in Fig. 2. The demands are related to both the number of evacuees and their time in the shelter, reflected in Fig. 2 as areas under the cumulative arrivals curve.

For non-consumable material, the criterion is to have supplies available at the shelters ahead of arrival of the evacuees, and a policy might be of the form: "By $t = 12$ h, have at least one-half of the total anticipated supplies in place, with the remainder available by $t = 24$ h." This policy also relates timing and quantity of demands for material to the forecast arrivals of evacuees at a shelter location in a particular event.

Because different events create varying numbers of evacuees in different locations, the demand for commodity $k$ at location $j$ in time period $t$, $v_{jt}^{k}$, is uncertain. This uncertainty is modeled through the use of a set $S$ of discrete scenarios indexed by $s \in S$, each with a probability of occurrence, $P_s$. The definition of a scenario includes the forecasted cumulative demand by commodity, location and time, $v_{jt}^{ks}$.

We define a set of time periods, $t = 1, 2,\ldots,T$. For example, we might use four time periods defined relative to the onset of activity: 0–12 h, 12–24 h, 24–48 h, and 48–72 h. The model does not require that the time periods considered be of equal duration.

We consider a set of commodities that may be pre-positioned in storage facilities and for which there is likely to be demand during and after an event. In the example application in Section 3, we consider two generic commodities ("consumables" – e.g., food, bottled water, etc.; and "non-consumables" – e.g., cots, blankets, etc.). This allows us to differentiate between commodities for which the demand is duration-dependent and those for which it is not. In a general application of this model, a larger set of individual commodities might be considered, including ice, clothing, medical supplies and other types of items. The set of commodities is denoted by $K$ and indexed by $k \in K$. In the aftermath of an event, there will be demands for these commodities at specific locations in a set $J$, indexed by $j \in J$. These locations will include designated shelters as well as selected other distribution points.

Supplies can be pre-positioned at locations indexed by $i$. Some of these locations (a set $I$) will correspond to places where a storage facility can be leased. Some shelter locations can also accommodate some storage of material, and we will use $I'$ to denote the set of shelter locations where material storage is possible. At the shelter locations $i \in I'$, there are capacity limits, $E_i$ on the total volume of material that can be stored. At locations that are not designated shelters, leasing a storage facility to make it available results in a fixed cost. For costing purposes, we define leased facilities to be in one of a discrete set, $L$, of size categories, indexed by $l \in L$. The overall capacity (e.g., square feet of floor space or cubic feet of total space) of a facility in category $l$ is $M_l$, and choosing to open a facility of size category $l$ in location $i \in I$ incurs a fixed cost, $F_{il}$. Let $y_{il}$ be a binary decision variable equal to 1 if there is a supply facility of capacity category $l$ located at node $i$, and 0 otherwise. This is one of the primary sets of decisions in the model.

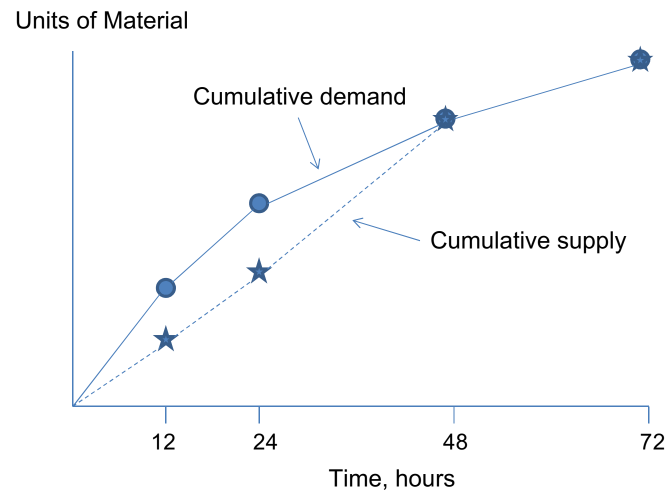


**Fig. 3.** Illustration of shortage occurrence.

If a facility is made available at location $i$, various commodities can be stocked there, subject to the capacity limits of the facility. Let $b^k$ be the unit space requirement for commodity $k$, and $r_i^k$ be the amount of commodity $k$ pre-positioned at location $i \in I \cup I'$. The $r_i^k$ values are the second major set of first-stage decision variables in the model. A decision to stock a particular commodity results in a unit acquisition cost, $q^k$. Material of type $k$ that is not used in scenario $s$, denoted $z_i^{ks}$, incurs an additional unit holding cost, $h^k$ (accounting for inventory-related or spoilage costs).

In each scenario, the optimization attempts to distribute available supplies of each commodity to meet demands at the various shelters and other locations. Let $c_{ij}^k$ be the unit cost of transporting commodity $k$ from source $i$ to destination $j$, and let $x_{ijt}^{ks}$ be the amount of commodity $k$ shipped in period $t$ for scenario $s$.

In addition to limited total amounts of each commodity available, there are three types of constraints on the shipment of available supplies. First, the storage locations are limited in the rate at which they can load and dispatch trucks. This rate is assumed to be related to the facility size, and be specified as a series of constants, $C_{tl}$, which specify the cumulative maximum amount that a facility of size class $l$ can dispatch by the end of period $t$. Second, there may be a lag between truck dispatch from storage location $i$ and receipt of the material at demand location $j$. We will denote this lag as $\eta_{ij}$. If storage location $i$ and demand location $j$ are quite close and we are using time periods of 12 or 24 h, there may be many pairs for which $\eta_{ij} = 0$ (i.e., the shipment arrives within the same period as dispatch). However, for locations that are more distant from one another, there may be a lag of one or more periods between dispatch and receipt. Third, transportation links may be damaged or destroyed in some scenarios, so we specify a scenario-dependent capacity, $U_{ijt}^s$, on transporting material from $i$ to $j$ at time $t$. The time-dependence of these capacity constraints allows the model to incorporate the possibility of some restoration of capacity over time for damaged links.

If not enough material of commodity $k$ can be supplied to location $j$ by time $t$ to meet the cumulative demand, a shortage occurs. Fig. 3 shows a situation where a shortage occurs in periods 1 and 2, which is alleviated by period 3. The area between the cumulative demand values and the cumulative arrival of material represents a total shortage, which is penalized in the objective function of the model. The difference between cumulative demand and delivered material at time $t$ is denoted as $w_{jt}^{ks}$. If we assume that the discrete points at periods $t$ are connected by straight lines (as in Fig. 3), the total shortage area for location $j$, commodity $k$ and scenario $s$ can be computed as a simple linear function of the $w_{jt}^{ks}$

values, $\sum \varphi_t w_{jt}^{ks}$, where the coefficients $\varphi_t$ depend on the number and duration of the time periods. These shortages can be converted to costs using penalty coefficients, $p^k$, in the objective function.

The model provides a confidence level (denoted $\alpha$) that sufficient supplies exist in the network to meet all demands. The idea of using a confidence level on meeting all demands draws on previous work by Daskin et al. [16] and Chen et al. [17]. Daskin et al. [16] introduced the idea of an "$\alpha$-reliable" location model, in which the user defines a set of scenarios, each with a probability of occurrence. The model then endogenously selects a subset of the scenarios (termed the *reliability set*) whose collective probability of occurrence is at least $\alpha$ and optimizes a measure of performance over that selected subset of scenarios. For Daskin et al. [16], the selected measure of performance was minimizing the maximum regret over scenarios in the reliability set. Chen et al. [17] extended this idea by minimizing the expected value of the regrets associated with the extreme scenarios (i.e., scenarios previously ignored by the $\alpha$-reliable minimax regret model) which have a collective probability of occurrence of $(1-\alpha)$.

We have adopted the idea of having the model endogenously select the reliability set of scenarios, but use that set in a different way. In our model, the objective function is an expected cost over all scenarios, but for scenarios in the reliable set, all demand must be satisfied (i.e., $w_{jt}^{ks} = 0$). A binary decision variable ($\gamma^s$) indicates if a scenario $s$ is included in the reliable set.

The model is formulated as an optimization problem (P1), in which the objective is to minimize the expected costs resulting from the selection of the pre-positioning locations and facility sizes, the commodity acquisition and the stocking decisions, the shipments of the supplies to the demand points, unmet demand penalties and holding costs for unused material.

$$\text{(P1)}\quad \min \sum_{i\in I}\sum_{l\in L} F_l y_{il} + \sum_{k\in K}\sum_{i\in I\cup I'} q^k r_i^k + \sum_{s\in S} P^s \left[ \sum_{i,j}\sum_{k\in K} \times \sum_t c_{ij}^{ks} x_{ijt}^{ks} + \sum_{i\in I\cup I'}\sum_{k\in K} h^k z_i^{ks} + p^k \sum_j \sum_k \sum_t \varphi_t w_{jt}^{ks} \right] \tag{1}$$

*Subject to:*

*Flow accounting*

$$z_i^{ks} = r_i^k - \sum_{j\in J}\sum_t x_{ijt}^{ks} \quad \forall\ i\in I\cup I', k\in K, s\in S \tag{2}$$

$$v_{jt}^{ks} - \sum_{i\in I\cup I'}\sum_{\tau=1}^{t-\eta_{ij}} x_{ij\tau}^{ks} - w_{jt}^{ks} \le 0 \quad \forall\ j\in J, k\in K, s\in S, t = 1,\ldots,T \tag{3}$$

$$\sum_{i\in I\cup I'}\sum_{\tau=1}^{T} x_{ij\tau}^{ks} \le v_{jT}^{ks} \quad \forall\ j\in J, k\in K, s\in S \tag{4}$$

*Open facilities*

$$\sum_{l\in L} y_{il} \le 1 \quad \forall\ i\in I \tag{5}$$

*Facility storage and dispatch capacity*

$$\sum_{k\in K} b^k r_i^k \le \sum_{l\in L} M_l y_{il} \quad \forall\ i\in I \tag{6}$$

$$\sum_{k\in K} b^k r_i^k \le E_i \quad \forall\ i\in I' \tag{7}$$

$$\sum_{\tau=1}^{t}\sum_{j\in J}\sum_{k\in K} u^k x_{ij\tau}^{ks} \le \sum_{l\in L} C_{tl} M_l y_{il} \quad \forall\ i\in I, s\in S, t = 1,\ldots,T \tag{8}$$

*Transport arc capacity*

$$\sum_{k\in K} u^k x_{ijt}^{ks} \le U_{ijt}^s \quad \forall\ i\in I, j\in J, s\in S, t = 1,\ldots,T \tag{9}$$

*Reliability set definition*

$$\sum_{s\in S} p^s \gamma^s \ge \alpha \tag{10}$$

*Demand requirements for scenarios in reliable set*

$$w_{jt}^{ks} \le v_{jt}^{ks}(1-\gamma^s) \quad \forall\ j\in J, k\in K, s\in S, t = 1,\ldots,T \tag{11}$$

*Binary constraints*

$$\begin{array}{ll} y_{il}\in(0,1) & \forall\ i\in I, l\in L \\ \gamma^s\in(0,1) & \forall\ s\in S \end{array} \tag{12}$$

*Non-negativity constraints*

$$\begin{array}{ll} r_i^k \ge 0 & \forall\ i\in I, k\in K \\ x_{ijt}^{ks} \ge 0 & \forall\ i\in I, j\in J, k\in K, s\in S, t = 1,\ldots,T \\ z_i^{ks} \ge 0 & \forall\ i\in I, k\in K, s\in S \\ w_{jt}^{ks} \ge 0 & \forall\ j\in J, k\in K, s\in S, t = 1,\ldots,T \end{array} \tag{13}$$

Constraint (2) defines the unused stocks in each scenario ($z_i^{ks}$). Constraint (3) defines the unmet demand ($w_{jt}^{ks}$), and constraint (4) restricts the solution from shipping extra material to avoid the holding costs on unused stocks. Constraint (5) limits the number of open facilities at node $i$ to one. Constraint (6) makes certain that stocked commodities at non-shelter locations are assigned to open facilities and that the space taken by these resources does not exceed the facility capacity. Constraint (7) limits the amount of material that can be stocked at the shelters. Constraint (8) represents the limits on the amount of material that can be shipped from a storage location by time $t$. Constraint (9) limits total $i$–$j$ flow within each time period to the capacity available in each scenario. A unit of commodity $k$ flowing from $i$ to $j$ requires capacity $u^k$. In many cases, the unit measures for commodity storage ($b^k$) and transport ($u^k$) may be the same, and only one value is necessary. However, the formulation allows for different measurement units, if desired. Constraint (10) defines the reliable set by determining which scenarios are included in the planning set. Constraint (11) ensures that demand is met for all scenarios included in the reliable set.

## 3. Case study application

As an illustration of the application of the model described in Section 2, we focus on meeting the demands for consumable and non-consumable goods in shelters for hurricane events that affect coastal North Carolina. Legg et al. [18] have created a set of hurricane scenarios and probabilities to match the historical regional hazard in North Carolina as closely as possible. Li et al. [19] in a study of evacuation and sheltering policies for North Carolina, used those scenarios to identify a set of 50 optimal shelter locations, selected from among a larger set of locations that are potential shelters. For each scenario, HAZUS-MH [20] is used to estimate the total number of people in each census tract that evacuate and seek public shelter. One of the results of the analysis done by Li et al. [19] is an estimate (by scenario) of the number of evacuees seeking shelter in each of the selected locations. We have used the same set

**Table 1**
Possible storage facility locations.

| Index | City | County |
|---|---|---|
| 1 | Sanford | Lee |
| 2 | Louisburg | Franklin |
| 3 | Smithfield | Johnston |
| 4 | Raleigh | Wake |
| 5 | Durham | Durham |
| 6 | Nashville | Nash |
| 7 | Gastonia | Gaston |
| 8 | Lincolnton | Lincoln |
| 9 | Charlotte | Mecklenburg |
| 10 | Fayetteville | Cumberland |
| 11 | Dunn | Harnett |
| 12 | Laurinburg | Scotland |
| 13 | Lumberton | Robeson |
| 14 | Raeford | Hoke |
| 15 | Hickory | Catawba |
| 16 | Statesville | Iredell |

**Table 2**
Hurricane demands and probabilities by scenario.

| Scenario | Probability | Demand | |
|---|---|---|---|
| | | Consumables | Non-consumables |
| 1 | 0.0127 | 219639 | 62,292 |
| 2 | 0.0101 | 171,663 | 48,686 |
| 3 | 0.0254 | 137,833 | 39,091 |
| 4 | 0.0101 | 132,471 | 37,569 |
| 5 | 0.0152 | 128,726 | 36,507 |
| 6 | 0.0025 | 100,977 | 28,641 |
| 7 | 0.0254 | 100,430 | 28,483 |
| 8 | 0.0178 | 79,578 | 22,570 |
| 9 | 0.0051 | 68,171 | 19,333 |
| 10 | 0.0584 | 66,558 | 18,875 |
| 11 | 0.0025 | 56,833 | 16,115 |
| 12 | 0.0152 | 50,244 | 14,249 |
| 13 | 0.0051 | 39,581 | 11,223 |
| 14 | 0.0482 | 37,356 | 10,595 |
| 15 | 0.0101 | 36,719 | 10,413 |
| 16 | 0.0025 | 35,267 | 10,002 |
| 17 | 0.0127 | 34,657 | 9828 |
| 18 | 0.1015 | 30,260 | 8581 |
| 19 | 0.0127 | 27,599 | 7825 |
| 20 | 0.0355 | 27,294 | 7740 |
| 21 | 0.0624 | 18,141 | 5145 |
| 22 | 0.0012 | 15,235 | 4320 |
| 23 | 0.0392 | 14,011 | 3974 |
| 24 | 0.0355 | 13,566 | 3848 |
| 25 | 0.0076 | 12,818 | 3636 |
| 26 | 0.0521 | 11,241 | 3188 |
| 27 | 0.1015 | 10,105 | 2865 |
| 28 | 0.0521 | 9373 | 2660 |
| 29 | 0.0051 | 8504 | 2411 |
| 30 | 0.0051 | 8189 | 2322 |
| 31 | 0.0533 | 7710 | 2185 |
| 32 | 0.0521 | 4583 | 1300 |
| 33 | 0.1041 | 3947 | 1120 |

of scenarios, probabilities, shelter locations and numbers of evacuees as input to our work.

The shelters include facilities with capacities between 750 and 3000 evacuees. However, each shelter is used in an average of 8 of the 33 scenarios, and most are only used to capacity in one or two of those events. The most frequently used shelters are activated in 17 of the 33 scenarios. Across the set of scenarios, the total number of evacuees ranges from 1018 to 56,630, with an average of 8637. The dynamics of the problem are modeled by assuming that 10% of the total evacuees at a given facility (in a specified scenario) arrive within the first 12 h, 33% arrive by $t = 24$ h, 90% arrive by $t = 48$ h, and all arrive by $t = 72$ h.

We use 16 counties as the potential locations for storage facilities for pre-positioned material, with the county seat of each county used as the point location. The cities and counties are listed in Table 1. The map in Fig. 4 shows the locations of the 50 shelter sites as well as the 16 potential storage facility locations.

We have converted the anticipated numbers of evacuees in each shelter location to a set of demands for consumable and non-consumable goods in each scenario. The resulting total demands and scenario probabilities are summarized in Table 2, ordered by decreasing overall demand. For non-consumable goods, we have assumed that a unit of demand is the amount of material for one person, and the total demand at a location is 1.1 times the number of anticipated evacuees sheltered there in a given scenario. Fifty percent of the total demand should be provided within the first 12 h (the first time period in the model), and the remainder should be provided within 24 h (the second time period). A person's worth of non-consumable goods is assumed to cost \$25, and incurs an additional holding cost of \$5.25 if unused. For storage and transportation purposes, we assume that the amount of space required for one person's worth of non-consumable material is 6 ft$^3$.

For consumable goods, the arrival pattern of evacuees is translated into a cumulative demand for person-days' worth of consumable materials, as illustrated in Fig. 5 (shown per 100 people in the shelter). This translation uses a 5% allowance for spoilage/waste, and assumes that the amount of material delivered within the first 72 h needs to be enough to sustain the entire evacuee population out to $t = 120$ h (5 days after onset of the event). A person-day of consumable goods is assumed to cost \$15, and incurs an additional holding cost of \$3.75 if unused. For the storage and

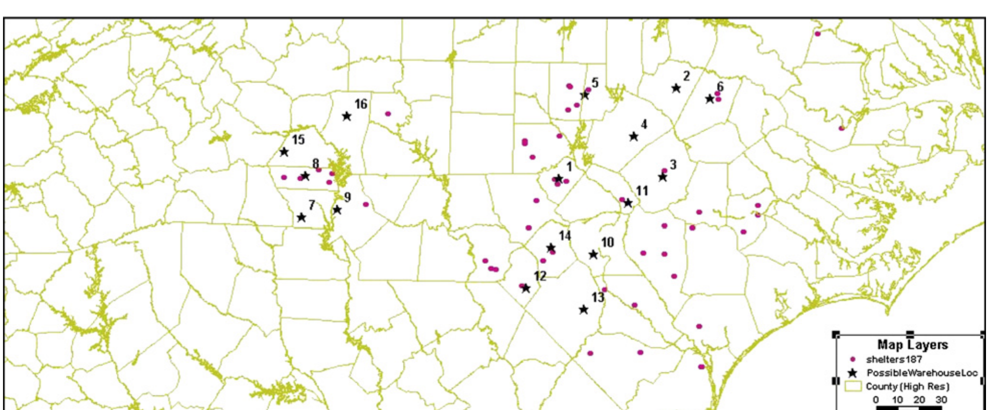


**Fig. 4.** Shelter and potential material pre-positioning locations.

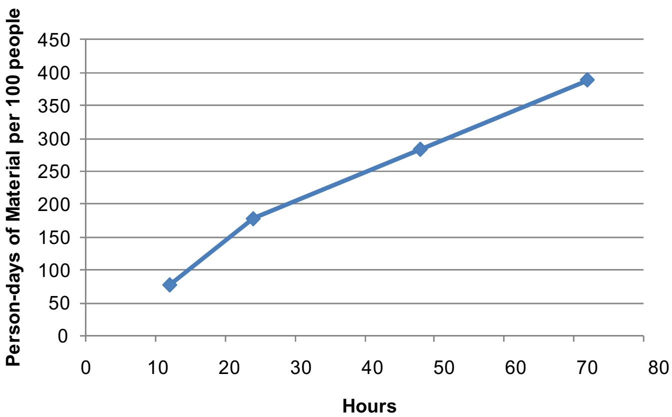


**Fig. 5.** Demand for consumables over four periods in the model.

**Table 3**
Parameters for potential storage facilities.

| Size Category | Descriptor | $F_l$ ($) | $M_l$ (ft$^3$) | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|---|---|---|
| 1 | Small | 20,000 | 30,000 | 0.5 | 1.0 | 1.0 | 1.0 |
| 2 | Medium | 48,000 | 100,000 | 0.15 | 0.4 | 0.7 | 1.0 |
| 3 | Large | 150,000 | 400,000 | 0.1 | 0.25 | 0.7 | 1.0 |

transportation calculations, one person-day of consumable goods is assumed to require 2 ft$^3$ of space.

The transportation costs for moving material are assumed to be $0.0015 per unit-mile for consumable goods and $0.0045 per unit-mile for non-consumable goods. The ratio of the transportation costs reflects the different assumptions of space requirements for a unit of consumables (one person-day's material) and a unit of non-consumables (one person's material). Distances for all origin–destination combinations reflect highway distances determined between Zip codes on the North Carolina network.

Origins and destinations that are less than 50 miles apart are assumed to allow transportation within the same time period. If locations are between 50 and 100 miles apart, it is assumed that there is a one-period transportation lag, and for locations that are more than 100 miles apart, the lag is assumed to be two periods. In this case study, the maximum distance between a potential storage facility and a shelter location is 310 miles.

Three possible sizes of facilities are considered, with parameters as shown in Table 3. A facility of any size can be opened at any of the 16 candidate locations. As a first experiment, we assume that the pre-positioning of supplies must satisfy the demands generated in all the scenarios (i.e., $\alpha = 1$). In order to meet the demand requirements in 100% of the cases, three facilities are opened (two large and one small), located in Sanford, Smithfield and Nashville. The location indices are listed in Table 4, from which the selected locations can be identified on the map in Fig. 4. A total of 219,639 units of consumable and 62,292 units of non-consumable supplies are stored. The facility and material acquisition cost (i.e., first-stage cost) for this solution is $5.17 million. The expected second-stage

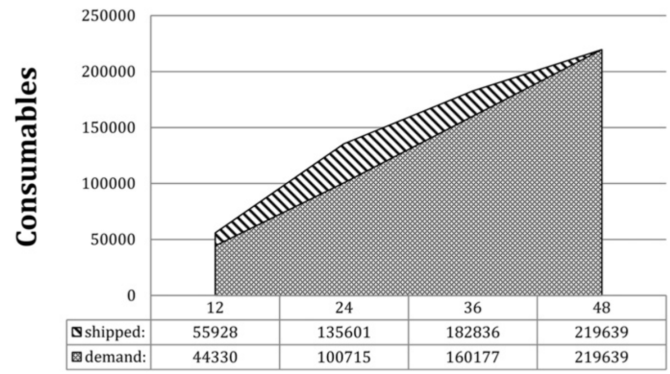


**Fig. 6.** Total consumable supplies needed and shipped in each time period during the first scenario for $\alpha = 1$.

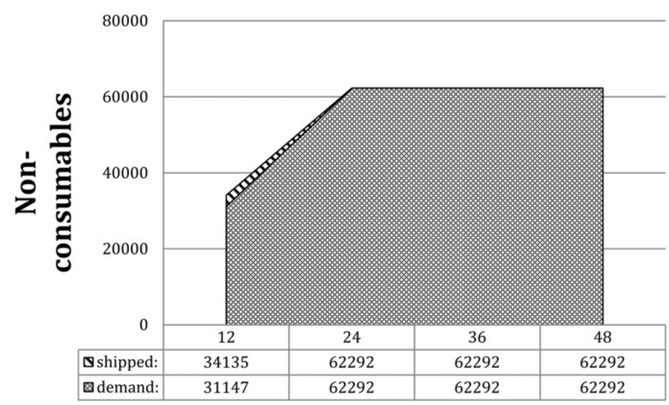


**Fig. 7.** Total non-consumable supplies needed and shipped in each time period during the first scenario for $\alpha = 1$.

cost (transportation and holding costs) is $1.03 million. There is no penalty cost for unmet demand in this case, because $\alpha = 1$. Although some storage of material is allowed at the shelter locations ($E_i$ values were set to 3000 ft$^3$ for the 25 largest shelters), none of this capacity is used in the optimal solution.

Figs. 6 and 7 show the quantities of consumables and non-consumables moved out of the distribution centers to the shelters over time for the first scenario in Table 2, which has the largest demands. Even in this scenario, the capacity of the distribution centers to move material to the shelters (as specified in Table 3) exceeds the demand rate, so the loading/transportation elements of this particular problem are not binding constraints on the solution.

If the reliability requirement is relaxed from 100% to 95%, the optimal solution is as summarized in Table 5. The number and locations of distribution centers are the same as when $\alpha = 1.0$, but the overall capacity is reduced by approximately 36%. The total first-stage costs decrease to $3.18 million. The expected second-stage costs are $1.37 million. The expected holding costs decrease by about $470,000 because less material is acquired in total, but

**Table 4**
Solution for $\alpha = 1.0$.

| Index | Facility locations | Facility capacity | Consumable supplies (units) | Non-consumable supplies (units) | Stored volume (ft$^3$) |
|---|---|---|---|---|---|
| 1 | Sanford | Large | 108,682 | 30,439 | 399,998 |
| 3 | Smithfield | Large | 102,557 | 29,653 | 383,032 |
| 6 | Nashville | Small | 8,400 | 2,200 | 30,000 |
| Total | | | 219,639 | 62,292 | |

**Table 5**
Solution for $\alpha = 0.95$ and $p^k = 20$ times purchase price.

| Index | Facility Locations | Facility capacity | Consumable supplies (units) | Non-consumable supplies (units) | Stored volume (ft$^3$) |
|---|---|---|---|---|---|
| 1 | Sanford | Large | 95,115 | 29,876 | 369,488 |
| 3 | Smithfield | Small | 15,000 | 0 | 30,000 |
| 6 | Nashville | Medium | 22,356 | 9,215 | 100,000 |
| Total | | | 132,471 | 39,091 | |

**Table 6**
Solution for $\alpha = 0.95$ and $p^k = 10$ times purchase price.

| Index | Facility locations | Facility capacity | Consumable supplies (units) | Non-consumable supplies (units) | Stored volume (ft$^3$) |
|---|---|---|---|---|---|
| 1 | Sanford | Large | 105,110 | 30,023 | 400,000 |
| 6 | Nashville | Medium | 27,361 | 7,546 | 100,000 |
| Total | | | 132,471 | 37,569 | |

**Table 7**
Solution for $\alpha = 0.95$ and $p^k = 50$ times purchase price.

| Index | Facility locations | Facility capacity | Consumable supplies | Non-consumables supplies | Volume |
|---|---|---|---|---|---|
| 1 | Sanford | Large | 89,077 | 36,974 | 399,998 |
| 3 | Smithfield | Medium | 21,626 | 9,458 | 100,000 |
| 6 | Nashville | Medium | 20,144 | 9,952 | 100,000 |
| 11 | Dunn | Small | 6,986 | 2,671 | 29,998 |
| Total | | | 137,833 | 59,055 | |

there are expected unmet demand penalties for the scenarios in the 5% of cases where demand is not fully met. Evaluation of these penalty costs depends on the coefficients $p^k$, and for this experiment we have set the penalty for each unit of unmet demand at 20 times the purchase price of the commodity. The total expected unmet demand penalty is approximately $802,000, associated with consumables in scenarios 1–3 and non-consumables in scenarios 1 and 2.

To test the sensitivity of the solution to the specification of the unmet demand penalty, we have also solved the $\alpha = 0.95$ problem with a lower penalty cost (10 times the purchase price of the commodities) and a higher penalty (50 times the purchase price). These solutions are summarized in Tables 6 and 7. When the penalty cost is reduced (as shown in Table 6), the solution has only two distribution centers and the total amount of non-consumable supplies stored is reduced. When the penalty cost is increased (as shown in Table 7), the optimal solution includes four facilities and the amount of consumable supplies stored is sufficient to meet demand in all but the two most severe scenarios (with total probability of occurrence 0.0228, as shown in Table 2). The amount of non-consumable material stored is sufficient for all but the worst case scenario (with probability 0.0127). In this case, the reliability constraint is no longer binding because the penalty cost of not meeting demand is high enough that the minimization of expected costs produces a solution that meets all demand in more than 95% of the possible outcomes.

Tables 5–7 illustrate an important basic property of the problem structure. The reliability constraint puts a “floor” under the amount of material stored, so that all demand can be met in the required proportion of outcomes, but if the penalty cost of not meeting demand is high enough, the optimal solution for stocking material may be above this level.

To investigate this further, a solution with $\alpha = 0.9$ and $p^k = 20$ times the material purchase price is shown in Table 8. This solution has sufficient capacity to meet all demands in at least 92.4% of the outcomes in Table 2, so the reliability constraint has been exceeded. Further reductions of the required reliability level with $p^k = 20$ times the material purchase price will have no effect on the solution because the cost tradeoff is driving the optimal solution.

At the lower level of penalty for unmet demand ($p^k = 10$ times material purchase price), the reliability constraint is active at lower levels because there is less incentive to purchase and store more material to avoid unmet demand. Tables 9 and 10 summarize the solutions for $\alpha = 0.9$ and $\alpha = 0.85$. The two solutions are very similar, but as the reliability constraint is relaxed, slightly less consumable material is stored. At $\alpha = 0.85$, the reliability constraint is not binding, so further reductions have no effect on the solution.

Although the solutions described above are based on minimizing total expected costs over all scenarios, in practice there may be special attention put on the first-stage costs of facility creation and stocking, because these are very visible costs of preparation for a disaster that may or may not occur. The numbers of facilities, total capacity and first-stage costs of the solutions summarized in Tables 5–10 are listed in Table 11. The range in capacity and first-stage costs of the solutions is approximately a factor of two, but solutions with relatively high reliability ($\alpha = 0.95$) are available at about 25% higher cost than the minimum solution.

Another interesting comparison is to contrast the solutions found with the dynamic allocation model (P1) to the simpler, static version of the model in Rawls and Turnquist [15]. We will focus on the solutions generated using an unmet demand penalty value of 20 times the supply purchase price. For the static model, we have assumed the same level of total demand at each shelter location as in the dynamic model, but there is no requirement about *when* supplies need to be delivered. The static model in [15] also contains no reliability requirement on the solution. The solution given by the static resource allocation formulation is summarized in Table 12. Although the static model in [15] does not include a reliability constraint, it is possible to compute that

**Table 8**
Solution for $\alpha = 0.9$ and $p^k = 20$ times purchase price.

| Index | Facility locations | Facility capacity | Consumable supplies (units) | Non-consumable supplies (units) | Stored volume (ft$^3$) |
|---|---|---|---|---|---|
| 6 | Nashville | Small | 15,000 | 0 | 30,000 |
| 11 | Dunn | Large | 70,715 | 39,091 | 375,976 |
| 12 | Laurinburg | Small | 15,000 | 0 | 30,000 |
| Total | | | 100,715 | 39,091 | |

**Table 9**
Solution for $\alpha = 0.9$ and $p^k = 10$ times purchase price.

| Index | Facility locations | Facility capacity | Consumable supplies (units) | Non-consumable supplies (units) | Stored volume (ft$^3$) |
|---|---|---|---|---|---|
| 6 | Nashville | Small | 15,000 | 0 | 30,000 |
| 11 | Dunn | Large | 85,430 | 37,569 | 400,000 |
| Total | | | 100,430 | 37,569 | |

**Table 10**
Solution for $\alpha = 0.85$ and $p^k = 10$ times purchase price.

| Index | Facility locations | Facility capacity | Consumable supplies (units) | Non-consumable supplies (units) | Stored volume (ft$^3$) |
|---|---|---|---|---|---|
| 6 | Nashville | Small | 15,000 | 0 | 30,000 |
| 11 | Dunn | Large | 78,877 | 37,569 | 383,168 |
| Total | | | 93,877 | 37,569 | |

**Table 11**
Cost and capacity summary.

| Reliability level ($\alpha$) | $p^k$ (multiple of material price) | Number of Facilities | Total capacity (ft$^3$) | First-stage costs (millions) |
|---|---|---|---|---|
| 1.0 | – | 3 | 830,000 | $5.17 |
| 0.95 | 50 | 4 | 630,000 | $3.80 |
| 0.95 | 20 | 3 | 530,000 | $3.18 |
| 0.95 | 10 | 2 | 500,000 | $3.12 |
| 0.9 | 20 | 3 | 460,000 | $2.68 |
| 0.9 | 10 | 2 | 430,000 | $2.62 |
| 0.85 | 10 | 2 | 430,000 | $2.52 |

**Table 12**
Static model solution for $p^k = 20$ times purchase price.

| Index | Facility locations | Facility capacity | Consumable supplies (units) | Non-consumable supplies (units) | Stored volume (ft$^3$) |
|---|---|---|---|---|---|
| 1 | Sanford | Large | 101,753 | 28,831 | 376,492 |
| 6 | Nashville | Medium | 26,973 | 7,676 | 100,000 |
| Total | | | 128,726 | 36,507 | |

the solution contains sufficient material to meet all demands with probability 0.94. Thus, the most direct comparisons are with the dynamic solutions for $\alpha = 0.9$ and $\alpha = 0.95$, as summarized in Tables 5 and 8.

The dynamic model solutions both specify three storage locations, rather than two. The dynamic solution for $\alpha = 0.95$ uses the same two locations as in the static solution, but adds a third (small) facility at Smithfield (location index 3). The total amount of material stored (both consumable and non-consumable) is a little larger in the dynamic solution. The dynamic solution for $\alpha = 0.9$ uses one of the two locations in the static solution (Nashville), but at a different size, and specifies two other locations for the other two facilities. That dynamic solution stores somewhat less consumable material and a little more non-consumables than called for in the static solution.

Addition of the reliability requirement (constraints 10 and 11 in problem P1) provides an important element in directing the solution. The static solution constructed without that requirement would not meet the $\alpha = 0.95$ reliability level. It would meet a requirement of $\alpha = 0.9$, but with a different selection of storage locations and material allocation than used in the optimal dynamic solution for $\alpha = 0.9$.

The addition of the dynamic demand requirements changes both the storage locations chosen and the material allocation, relative to the static solution. These changes are important to ensure that sufficient material can be supplied to the shelters quickly, and increase the quality of the service provided.

Computation of the solutions described here has been done with the CPLEX C++ library on a Linux X86-64 workstation with dual quad-core processors, a clock speed of 1.86 GHz and 8 GB of memory. CPLEX uses all eight cores in parallel during the solution process. The solution times range from about 10 min to more than 47 h, depending on the values of $\alpha$ and $p^k$. The shortest solution time (by far) is for the case where $\alpha = 1.0$ because there are no decisions to be made about excluding scenarios from the reliability set and there are no unmet demands to be evaluated. Most of the solutions required 6–11 h of computation time.

## 4. Conclusions and further directions

This paper presents a dynamic allocation model that optimizes pre-event planning for meeting short-term demands (over approximately the first 72 h) for emergency supplies at shelter locations due to hurricane threats. The model includes uncertainty about what demands will have to be met and where those demands will occur. The model also includes requirements for reliability in the solutions ensuring that all demands are met in scenarios comprising at least 100$\alpha$% of all outcomes. A case study using shelter locations in North Carolina and a set of hurricane threat scenarios is used to illustrate application of the model and how it supports an emergency relief strategy.

The model presented is a two-stage stochastic program that offers a basis for planning for the arrivals of evacuees at shelter locations. For consumable materials, the model follows the policy of placing enough material at the shelter location within the first 12 h of a hurricane threat to handle anticipated needs over the first 48 hours; and by the end of each day there should be enough material on hand to handle the next two days of operations without disrupting service while emergency conditions prevail. For non-consumable materials, the plan requests that all supplies arrive within the first 24 h of emergency.

A series of experiments was conducted to determine the influence of the reliability parameter ($\alpha$) on the solutions and the interactions between $\alpha$ and the penalty cost of unmet demand ($p^k$). The reliability requirement places a “floor” under the required material stocked, but when the penalty for unmet demand is high enough, the reliability constraint no longer affects the solution. This interaction is an important part of evaluating potential solutions.

The long computational times for solutions with $\alpha < 1.0$ suggest the value of developing a specialized algorithm to solve this dynamic resource allocation problem. Several possible approaches to this problem are currently being considered.

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