Satellite Distribution Centers

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CONTENTS

01 : **Big Picture**

What Happened?

In the Canadian province of Quebec, the Civil Protection Act (CPA) was adopted by the government and went into effect on December 20, 2001. According to this CPA, each municipality must develop and update its own emergency preparedness plan, which includes all topics related to emergency logistics.

The objective of the article is to provide a tool that supports the emergency managers in designing and operating a satellite distribution center network.

02: **Mathematical Model**

Assumptions and Parameters

- $G = (V, A)$ be a complete directed graph in which V represents the vertices and A is the arc set.
  $V = \{0\} \cup I \cup J$ where 0 is the central depot, $I = \{1, \ldots, n\}$: the set of demand points.
  $J = \{1, \ldots, m\}$: the set of potential satellite distribution centers
  $A = \{(v_i, v_j): v_i, v_j \in V\}$
- C_{ij}: distance matrix defined on A
- d_{is}: The amount of aid of type s ($s = 1, \ldots, t$) required at demand point $i \in I$
- w_s: weight of each aid unit s
- Q_k: the capacity (in units) of vehicle $k=1, \ldots, l$.
- α : a n*m matrix, in which α_{ij} is equal to 1 if demand point i is within the covering distance s from SDC j, and 0, otherwise.
- All the demand points of I must be covered.

02 : **Mathematical Model**

Decision Variables

D_{isjk}	quantity of demand type s at demand point i supplied by vehicle k while visiting SDC j;
x_{ijk}	equals 1 if arc (i, j) is used by vehicle k, and 0, otherwise;
y_{jk}	equals 1 if SDC j is visited by vehicle k, and 0, otherwise; and
u_{ik}	a free variable used in the sub-tour elimination constraints.

02: Mathematical Model

$$Min \sum_{i=0}^{m} \sum_{j=0}^{m} \sum_{k=1}^{l} c_{ij} x_{ijk} \quad (1)$$

s. t. :

$$\sum_{i=0}^{m} x_{ijk} = y_{jk} \quad j \in \{1, 2, \ldots, m\}, k \in \{1, 2, \ldots, l\} \quad (2)$$

$$\sum_{i=0}^{m} x_{jik} = y_{jk} \quad j \in \{1, 2, \ldots, m\}, k \in \{1, 2, \ldots, l\} \quad (3)$$

$$\sum_{j=0}^{m} x_{0jk} = 1 \quad k \in \{1, 2, \ldots, l\} \quad (4)$$

$$\sum_{j=0}^{m} x_{j0k} = 1 \quad k \in \{1, 2, \ldots, l\} \quad (5)$$

02: Mathematical Model

$$\sum_{j=1}^{m}\sum_{k=1}^{l}\alpha_{ij}D_{isjk} \geqslant d_{is} \quad i \in \{1,2,\ldots,n\}, s \in \{1,2,\ldots,t\} \tag{6}$$

$$\sum_{i=1}^{n}\sum_{s=1}^{t}w_s D_{isjk} \leqslant Q_k y_{jk} \quad k \in \{1,2,\ldots,l\}, j \in \{1,2,\ldots,m\} \tag{7}$$

$$\sum_{s=1}^{t}\sum_{i=1}^{n}\sum_{j=1}^{m}w_s D_{isjk} \leqslant Q_k \quad k \in \{1,2,\ldots,l\} \tag{8}$$

02: **Mathematical Model**

subtour elimination constraint

$$u_{ik} - u_{jk} + (m+1)x_{ijk} \leqslant m \quad i,j \in \{1,2,\ldots,m\}, k \in \{1,2,\ldots,l\} \quad (9)$$

considering that $\sum_{s=1}^{t}\sum_{h=1}^{n} w_s D_{hsjk}$ is the demand of SDC j:

$$u_{ik} - u_{jk} + Q_k x_{ijk} \leqslant Q_k - \sum_{s=1}^{t}\sum_{h=1}^{n} w_s D_{hsjk} \quad i,j \in \{1,2,\ldots,m\} \quad k \in \{1,2,\ldots,l\} \quad (14)$$

one central depot,
13 potential satellite distribution centers
42 demand points

Solution : 2 vehicle routes
5 satellite distribution centers

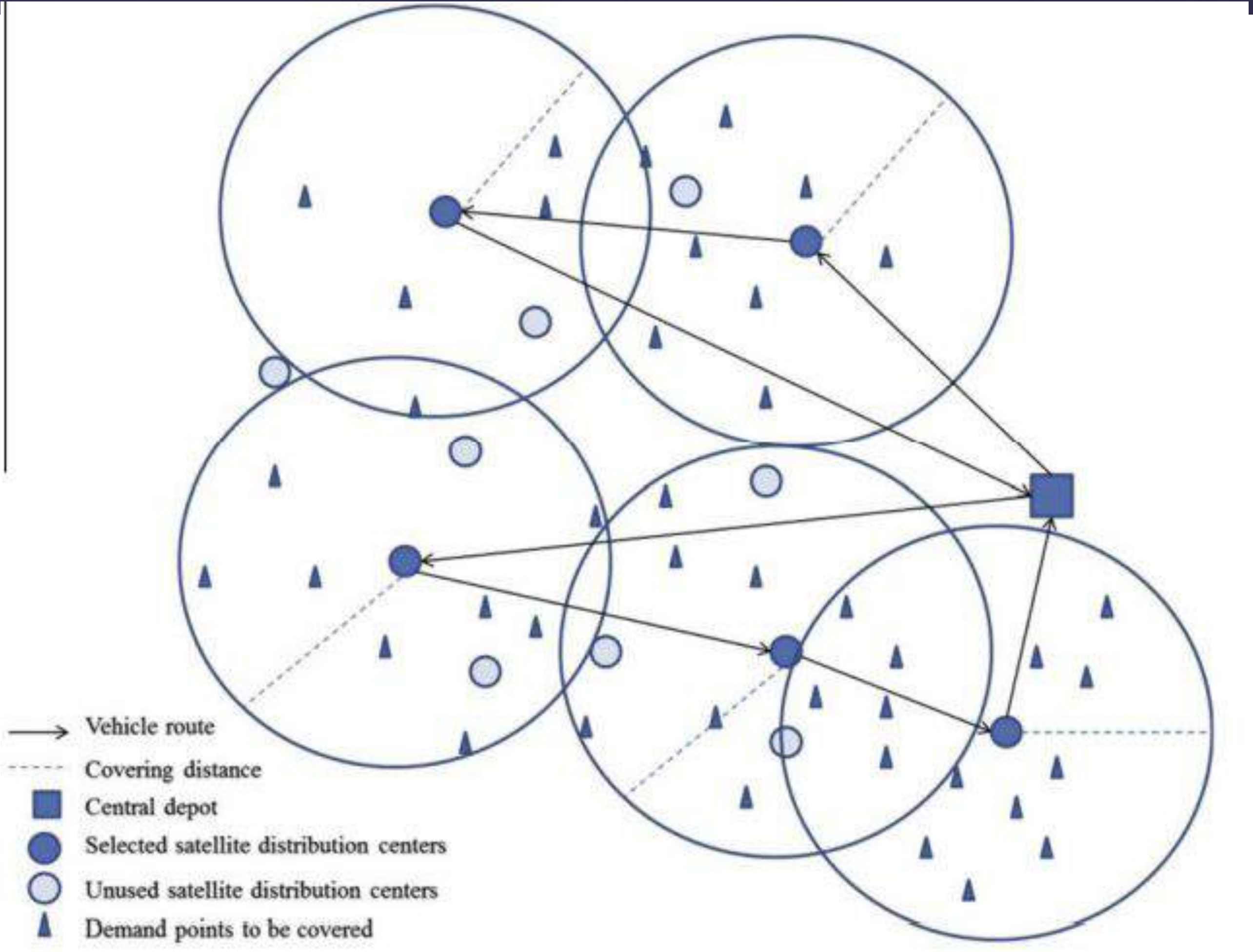

Fig. 1. Example of the studied network.

03 Additional Example

Table 1
Characteristics of major related problems.

Problem name	Authors	Objective function	No. of vertices in the subtour	Kinds of node	Covering distance	Nodes with demand	Nodes with profit	No. of products	No. of vehicles
Covering Salesman Problem – CSV	Current and Schilling (1989)	Minimize distance	Fixed, p	One	Yes	No	No	1	1
Median Tour Problem – MTP	Current and Schilling (1994)	Minimize Z_1 = distance and Z_2 = assignment cost	Fixed, p	One	No	Yes	No	1	1
Maximal Covering Tour Problem – MCTP	Current and Schilling (1994)	Minimize Z_1 = distance and maximize and Z_2 = demand within a covering distance	Fixed, p	One	Yes	Yes	No	1	1
Covering Tour Problem – CTP	Gendreau et al. (1997) and Baldacci et al. (2005)	Minimize distance while covering nodes of W_2	Free	W_1 can be visited, some must be visited and W_2 must be covered	Yes	No	No	1	1
Bi-Objective Covering Tour Problem – BOCTP	Jozefowiez et al. (2007)	Minimize distance of visited nodes and the maximum distance of covered nodes	Free	W_1 can be visited, some must be visited and W_2 must be covered	Yes	No	No	1	1

Multi-Objective Covering Tour Problem – MOCTP	Nolz et al. (2010)	Combination of two objectives choosen between three	Free	One	Yes	Yes	No	1	m
Multi-Vehicle Covering Tour Problem – m-CTP	Hachicha et al. (2000)	Minimize distance while covering nodes of W_2, subject to maximum number of nodes and maximum distance per route	No more than p	W_1 can be visited, some must be visited and W_2 must be covered	Yes	No	No	1	m
Prize Collecting Traveling Salesman Problem – PCTSP	Fischetti and Toth (1988)	Minimize distance subject to a minimum profit collected	Free	One	No	No	p_i	1	1
Selective Traveling Salesman Problem – STSP	Laporte and Martello (1990)	Maximize profit subject to a maximum distance	Free	One	No	No	p_i	1	1
Median Cycle Problem – MCP1	Moreno Pérez et al. (2003), Kedad-Sidhoum and Hung Nguyen (2010), and Renaud et al. (2004)	Minimize distance and assignment cost	Free	One	No	No	No	1	1
Median Cycle Problem – MCP2	Moreno Pérez et al. (2003) and Renaud et al. (2004)	Minimize distance subject to a maximum assignment cost	Free	One	No	No	No	1	1
Current contribution		Minimize distance	Free	J can be visited and I must be covered	Yes	Yes	No	Many	m

04 : **Heuristic Implemented**

General Outlook

Buraya export ettikten sonra koyucam

Tamam

01 : **Overview**

What Happened?

- The US, the UK and Germany were the top donors to the international Ebola response, donating more than $3.611 billion by December 2015.
- The US government allocated $2.369 billion for Ebola response activities in Guinea, Liberia, and Sierra Leone.

5: **Computational Results**

This section has three objectives:

1) Identifying parameter combination that produces best results
2) Identifying limits of the mathematical model
3) Evaluate the quality of heuristic approach in terms of computational time and objective function

5: Computational Results

- A set of numerical experiments are built based on randomly-generated data.
- The instances are characterized by the number of demand points (n), the number of potential SDCs (m), the number of different products (t) and the number of vehicles available (l).
- For each DP and potential SDC, the coordinates were uniformly generated within a [0,100] square.
- Distances between each pair of sites c_{ij} are linear distances.

5: **Computational Results**

Setting the heuristics parameters:

20 instances with n=100 demand points, m=20 SDCs, t=2 product types, l=2 vehicle types and 3000 restarts

- Number of iterations (#_iterations) : 10, 20
- Number of local search iterations (#_LocalSearch_iterations): 10, 15, 20
- Number of SDCs considered in the swap (ϕ): 4, 6, 8
- Number of SDCs considered in the diversification (φ): 3, 5, 7

5: Computational Results

Table 2
Heuristic parameters setting.

#_iterations	#_LocalSearch_iterations	φ	ϕ		
			4	6	8
10	10	3	1283.95	1274.45	1277.35
		5	1278.50	1272.15	1268.90
		7	1284.30	1276.00	1274.20
	15	3	1285.25	1274.20	1273.45
		5	1280.35	1279.45	1270.85
		7	1277.00	1273.75	1275.95
	20	3	1283.85	1277.80	1279.20
		5	1278.55	1278.55	1271.85
		7	1279.35	1276.90	1273.75
20	10	3	1288.15	1270.10	1272.35
		5	1277.30	1270.10	*1265.00*
		7	1278.45	1278.75	1276.55
	15	3	1280.70	1277.50	1274.70
		5	1279.40	1277.70	1277.20
		7	1286.05	1273.20	1272.40
	20	3	1273.05	1282.45	1272.00
		5	1270.70	1277.60	1270.40
		7	1277.30	1279.10	1272.60

The numbers in italics correspond to the best average solution.

5: Computational Results

- Some other instance sets are generated in order to establish independence from the instances used to calibrate our parameters.
- Four categories of vehicles are considered. Their capacities are {50, 75, 100, 150} units, respectively.
- The instances with different vehicle types (l = 2, l = 3 and l = 4) have vehicle capacities of {50, 75}, {50, 75, 100}, and {50, 75, 100, 150} units, respectively.
- For a given instance, the vehicles are added until their total capacity is equal or greater than the total demand, multiplied by a factor of 1.2

Table 3
Numerical results for the small instances ($n = 20$ DPs).

Set	SDC	Products	Vehicles	Exact		Heuristic		
				Cost	Seconds	Cost	Gap (%)	Seconds
1	4	2	2	284.00	0.31	284.00	0.00	3.29
2	4	2	3	226.20	0.22	226.20	0.00	3.30
3	4	2	4	212.60	0.57	212.60	0.00	3.43
4	4	3	2	464.80	8.57	465.60	0.17	6.51
5	4	3	3	417.60	1.98	421.60	0.96	5.29
6	4	3	4	313.60	1.37	317.00	1.08	4.05
7	4	4	2	813.60	52.58	813.60	0.00	12.19
8	4	4	3	628.00	19.30	628.00	0.00	9.12
9	4	4	4	497.40	5.08	497.40	0.00	6.79
10	6	2	2	208.60	5.22	208.60	0.00	4.59
11	6	2	3	183.80	8.77	183.80	0.00	4.62
12	6	2	4	134.40	2.89	134.40	0.00	4.32
13	6	3	2	435.60	100.87	435.60	0.00	8.03
14	6	3	3	347.40	46.48	347.40	0.00	6.73
15	6	3	4	265.80	10.23	265.80	0.00	5.88
16	6	4	2	694.40	734.94	694.40	0.00	15.20
17	6	4	3	564.00	746.87	564.00	0.00	12.26
18	6	4	4	444.20	266.69	444.20	0.00	9.79
19	8	2	2	301.40	367.82	301.40	0.00	5.49
20	8	2	3	265.40	363.98	265.40	0.00	5.11
21	8	2	4	228.60	31.01	228.60	0.00	5.08
22	8	3	2	430.60	783.29	430.60	0.00	10.06
23	8	3	3	356.00	469.30	355.80	*−0.06*	8.84
24	8	3	4	286.80	313.87	286.80	0.00	8.04
25	8	4	2	722.80	942.80	713.40	*−1.30*	15.12
26	8	4	3	542.80	805.38	538.20	*−0.85*	12.16
27	8	4	4	432.20	731.40	432.20	0.00	10.00
28	10	2	2	265.40	364.25	265.40	0.00	5.09
29	10	2	3	230.20	42.76	230.20	0.00	5.25
30	10	2	4	218.60	41.76	218.60	0.00	5.72
31	10	3	2	355.00	1180.55	354.80	*−0.06*	9.95
32	10	3	3	296.00	426.89	296.60	0.20	8.31
33	10	3	4	245.80	488.41	245.80	0.00	8.09
34	10	4	2	624.40	1672.34	616.40	*−1.28*	14.31
35	10	4	3	517.60	1156.36	516.60	*−0.19*	11.70
36	10	4	4	406.60	813.56	406.60	0.00	9.72
Average					361.35		−0.11	7.87
Minimum					0.22		−1.30	3.29
Maximum					1672.34		1.08	15.20

5: Computational Results

- For these small instances, Cplex was allowed to run for up to 1800 s. Within this time limit, it was able to give proof of optimality in 152 out of 180 cases.
- Table 3 confirms the excellent performance for the heuristic. For 26 out of 36 sets, the heuristic average gap was 0%, for the five instances in each of these 26 sets, the heuristic found the best known solutions.
- For the other six sets, the average gap of the heuristic was negative, meaning that the heuristic produces a better solution than Cplex in the allotted time. The heuristic average gap is - 0.11, with an average computing time of 7.8 s.

5: Computational Results

- In Tables 4–6,the larger instance results are reported in order to evaluate the ability of heuristic to solve real problems efficiently.
- 24 new sets are generated of five instances each, with different combinations of numbers of DP, SDC, products and vehicle types.
- The instances which had up to 50 DP, 20 SDC, four products and four vehicle types were solved by running Cplex for up to 7200 s for each instance.
- For these new 120 instances, Cplex was only able to find eight proven optimal solutions, all of them for n = 30 DP instances. (Table 4)

5: Computational Results

Table 4
Results for instances with $n = 30$ DP.

Set	SDC	Products	Vehicles	Exact			Heuristic		
				Cost	Gap (%)	Seconds	Cost	Gap (%)	Seconds
1	9	3	3	528.20	14.45	4924	526.40	–0.34	17.72
2	9	3	4	925.60	34.83	5829	906.00	–2.12	31.53
3	9	4	3	421.40	7.37	3127	421.20	–0.05	15.14
4	9	4	4	665.80	28.31	5784	675.60	1.47	24.29
5	12	3	3	504.80	28.49	7200	503.20	–0.32	18.83
6	12	3	4	903.80	50.90	7200	857.20	–5.16	32.31
7	12	4	3	419.20	17.90	6002	418.00	–0.29	15.97
8	12	4	4	691.60	39.02	7200	662.60	–4.19	24.92
Average					27.66	5908		–1.37	22.59

Table 5
Results for instances with $n = 40$ DP.

Set	SDC	Products	Vehicles	Exact			Heuristic		
				Cost	Gap (%)	Seconds	Cost	Gap (%)	Seconds
1	12	3	3	655.80	47.58	7200	632.20	–3.60	31.76
2	12	3	4	1191.80	59.86	7200	1120.80	–5.96	54.74
3	12	4	3	503.60	53.34	7200	487.20	–3.26	26.90
4	12	4	4	1011.80	57.81	7200	961.00	–5.02	46.34
5	16	3	3	652.60	58.88	7200	478.00	–26.75	30.75
6	16	3	4	976.00	65.74	7200	931.00	–4.61	60.65
7	16	4	3	463.60	46.78	7200	384.00	–17.17	26.55
8	16	4	4	791.80	64.89	7200	769.00	–2.88	50.29
Average					58.86	7200		–8.66	41.00

5: Computational Results

Table 6
Results for instances with $n = 50$ DP.

Set	SDC	Products	Vehicles	Exact			Heuristic		
				Cost	Gap (%)	Seconds	Cost	Gap (%)	Seconds
1	12	3	3	843.60	67.35	7200	775.80	−8.04	50.00
2	12	3	4	660.60	63.70	7200	582.20	−11.87	41.89
3	12	4	3	1432.00	75.01	7200	1350.40	−5.70	90.86
4	12	4	4	1089.60	73.08	7200	996.00	−8.59	69.22
5	16	3	3	807.80	71.23	7200	720.80	−10.77	57.29
6	16	3	4	627.00	63.74	7200	561.40	−10.46	49.10
7	16	4	3	1310.00	81.67	7200	1146.40	−12.49	88.42
8	16	4	4	984.80	79.77	7200	861.40	−12.53	72.74
Average					71.94	7200		−10.06	64.94

6 : Alternative Actions

- Different areas of a country are prone to different types of disasters.
- The current model does not take into account the fact that in case of disasters, some roads may not be safe to use and may become damaged.
- Some of these roads may be in the path of a SDC, and so even if they are within the threshold value, they may not be available to use.

6 : Alternative Actions

New Parameters:

q_a : probability that disaster type a will occur a = 1,…,A

p_{aij} : probability that the road from demand point i to SDC j will be inaccessible because of disaster type a a = 1,…,A ; i = 1,…,n ; j = 1,…,m

Change in Objective Function

$$Min \sum_{i=0}^{m} \sum_{j=0}^{m} \sum_{k=1}^{i} \sum_{a=1}^{A} q_a * p_{aij} * C_{ij} * X_{ijk}$$

7 : Comparison in Turkey

Vehicle Routing Problem (VRP) of a Logistics Firm in Turkey:

- The mathematical model used for the transportation activity in accordance with available data for one storage for 16 different customers.

  Parameters:

  K : Total vehicle number

  N : Total customer number

  Cij : Transportation cost from source i to destination j

  C0 : Cost of holding one unit of product on stock for one day

  Mi : Customer demand on i

7 : Comparison in Turkey

Indexes:
i : customer point i
j : customer point j
s : customer point i or j

Positive Variables:
Tki : Meeted customer demand at point i
Ei : Backlogged customer demand at point i

0-1 Variable: Xijk : if transport k travels from point i to j, then 1, else 0

7 : Comparison in Turkey

(I) Objective function:

$$Min\ Z = \sum_{i=0}^{16} \sum_{j=0, i \neq j}^{16} \sum_{k=1}^{20} C_{ij}\, x_{ijk} + \sum_{i=1}^{16} E_i\, C_0$$

- The aim of the objective function is minimizing the transportation cost and holding in stock cost.

7 : Comparison in Turkey

$$\sum_{i=1}^{16} T_{ki} \leq 17000 \qquad k=\{1,\ldots,16\}$$

- According to Constraint (2), the satisfied demand of each customer with one vehicle should be equal or less than 17000.

7 : Comparison in Turkey

(3) Constraint of incoming branch being higher than outgoing branch:

$$\sum_{i=0, i\neq s}^{16} \sum_{k=1}^{20} x_{isk} \geq \sum_{j=1}^{16} \sum_{k=1}^{20} x_{sjk} \qquad s=\{1,\ldots 16\}$$

According to Constraint (3) total number of incoming branch must be more than that of outgoing branch. Since the vehicles start their route from stock. However, they do not return back and they stay their last destination. Therefore, there is one branch arriving in the node and no branch going out the node.

7 : Comparison in Turkey

(4) Constraint of each vehicle starting to shuttle from stock to route:

$$\sum_{j=1}^{16} x_{0jk} \geq \sum_{i=0}^{16} \sum_{j=1, j\neq i}^{16} x_{ijk} \qquad k=\{1,,,,20\}$$

- Constraint (4) shows that the route start point is the stock for each vehicle.

7 : Comparison in Turkey

(5) Constraint of Demand:

$$\sum_{k=1}^{20} T_{ki} + E_i = M_i \qquad i=\{1,....,16\}$$

- According to Constraint (5), a customer demand consists of the satisfied demand and unsatisfied demand, which implies the stock.

7 : Comparison in Turkey

(6) Constraint of having enough routes for demand:

$$\sum_{i=0}^{16} \sum_{j=1, i \neq j}^{16} x_{ijk} . M \geq \sum_{i=1}^{16} T_{ki} \qquad k = \{1, \ldots, 20\}$$

- Constraint (6) demonstrates that there must be enough routes for satisfying the demand. The number M is used instead of a very large number.

7 : Comparison in Turkey

(7) Constraint of returning from another way:

$$x_{ijk} + x_{jik} = 1$$

(8) Constraint of integer

$$x_{ijk} \in \{0,1\}$$

- According to Constraint (7), the vehicle does not keep going on its route by retracing its steps. In other words, if the vehicle goes from customer i to customer j, it does not return to customer j from customer i.
- Constraint (8) shows that Xijk can take the value of 0 or 1.

References

1. Z. Naji-Azimi, J. Renaud, A. Ruiz, M. Salari, "A Covering Tour Approach to the Location of Satellite Distribution Centers to Supply Humanitarian Aid," *European Journal of Operational Research*, May 2012. [Online]. Available: https://courses.ie.bilkent.edu.tr/ie482/wp-content/uploads/sites/3/2019/04/Article-16_Satellite-Distribution-Center.pdf [Accessed: 10 April, 2019].
2. I. Uyan, M. Oturakçı, "The Optimization of a Vehicle Routing Problem in a Logistics Company in Turkey," *Alphanumeric Journal,* July 2014. [Online]. Available: http://dergipark.gov.tr/download/article-file/19230. [Accessed: 14 April, 2019].

THANK YOU FOR YOUR LISTENING

ANY QUESTIONS ?