Development Logistics Example

Urban Healthcare: Periodic Location Routing Problem

Sinem Savaser, Bahar Yetis Kara



Healthcare in Rural Areas

Lack of sufficient healthcare services in rural areas has been a considerable problem throughout the world.



(T)

Healthcare Issues in Turkey

Urban Areas

- Death rate among new born babies: 1.6%
- Vaccination rate until age of
 2: 74%
- Medical assistance in births:
 91%

Rural Areas

- Death rate among new born babies: 3.9%
- Vaccination rate until age of
 2: 60%
- Medical assistance in births: 74%

Mobile Healthcare Services

Possible Solutions

- Encourage doctors with privileges and promotions
- More investments on medical centers
- Mobile healthcare services

Mobile Healthcare Services

Transportation of medical staff to the villages without any medical centers

Applications in the world Since 2010 in Turkey

10 villages to a family practice center

Problem Specific Requirements

- Visiting frequencies depend on the population size.
- There are alternative visiting rules for each frequency level.

Population Size	Minimum Visiting Hours (per month)	Frequencies (half-day/month)	Visiting Rule Alternatives
\leq 100	4	1	1 half-day in a month
\leq 300	8	2	1 day in a month 1 half-day in each two weeks
\leq 750	16	4	1 day in each two weeks 1 half-day in each week
≤ 1000	32	8	1 day in each week 1 half-day in each 2.5 days
> 1000	48	12	1.5 days in each week

Requirements of the Problem

• Services must be provided at the same slot each week.



Problem Definition

- Generate monthly service schedules for the practitioners to travel the villages,
- Determine their base hospitals,
- While satisfying problem specific requirements.
 - Over the population of the population size.
 - **2** There are alternative visiting rules for each frequency level.
 - Services must be provided at the same slot each week.
 - Octors are dedicated to the villages.
 - Base hospitals for each doctor must be selected (where they start their tour from on Monday morning and end them on Friday afternoon)

Periodic Location Routing Problem

Variations of Classical Routing Problems

- Distance Constrained
- Time Windowed
- Multi-depot
- Split Delivery
- Heterogenous Fleet
- Pick-up and delivery together

Periodic

Periodic Location and Routing Problem (PLRP)

Location(s) of the depot fixed or to be determined

Periodic Location and Routing Problem

- Determine periodic routes
- Determine the location of the depot
- The literature on PLRP is scarce
- Visit the Periodic Vehicle Routing Problem literature
 Depot location is fixed

Mathematical Model

Sets:

N	Set of all nodes, $N = I \cup H$.
Ι	Set of villages.
I2, I4, I8, I12	Set of villages with frequency 2, 4, 8, 12, respectively.
H	Set of hospitals.
D	Set of doctors (practitioners).
T	Set of time periods.
NT1	Set of time periods consisting of $\{11, 21, 31\}$
NT01	Set of time periods consisting of $\{10, 11, 20, 21, 30, 31\}$

Parameters:

$DIST_{nm}$:	distance between nodes $n \in N$ and $m \in N$.
DEM_i :	visiting frequency of village $i \in I$.
CAP:	maximum working time of doctors.
p:	number of base hospitals to be selected.

Mathematical Model

Decision Variables

 $x_{nm}^{dt} = \begin{cases} 1, & \text{if doctor } d \in D \text{ travels from node } n \in N \text{ to } m \in N \text{ at time period} \\ & t \in T, \\ 0, & \text{otherwise.} \end{cases}$ $y_i^{dt} = \begin{cases} 1, & \text{if doctor } d \in D \text{ visits village } i \in I \text{ at time period } t \in T, \\ 0, & \text{otherwise.} \end{cases}$ $u_n^d = \begin{cases} 1, & \text{if node } n \in N \text{ is assigned to doctor } d \in D, \\ 0, & \text{otherwise.} \end{cases}$ $z_h = \begin{cases} 1, & \text{if a hospital at } h \in H \text{ is selected as a base hospital,} \\ 0, & \text{otherwise.} \end{cases}$ $k_{ih}^{dt} = \begin{cases} 1, & \text{if doctor } d \in D \text{ who is assigned to the hospital at point } h \in H \\ & \text{is present at village } i \in I \text{ at time period } t \in T, \\ 0, & \text{otherwise.} \end{cases}$

Mathematical Model (Routing Decisions)

 $\sum \sum \sum x_{nm}^{dt} \cdot DIST_{nm} - \sum \sum \sum x_{nm}^{dt} \cdot DIST_{nm}$ minimize (1) $n \in N \ m \in N \ d \in D \ t \in T$ $n \in N \ m \in N \ d \in D \ t \in NT1$ $+\sum\sum\sum\sum \sum k_{ih}^{dt} \cdot DIST_{ih},$ $i \in I$ $h \in H$ $d \in D$ $t \in NT01$ subject to $\sum \sum x_{hi}^{d1} = 1,$ $d \in D$ (2) $i \in I \quad h \in H$ $\sum \sum y_i^{dt} = DEM_i,$ $i \in I$ (3) $d \in D t \leq 40$ $\sum x_{ni}^{dt} = y_i^{dt},$ $i \in I, d \in D, t \leq 40$ (4) $n \in N$ $\sum x_{in}^{dt+1} = y_i^{dt},$ $i \in I, d \in D, t \leq 40$ (5) $n \in N$ $y_i^{dt} \leq u_i^d$, $i \in I, d \in D, t \leq 40$ (6) $\sum u_i^d = 1,$ $i \in I$ (7) $d \in D$

Mathematical Model (Routing Decisions)

$\sum_{i \in I} y_i^{dt} \le 1,$	$d \in D, t \leq 40$	(8)
$\sum_{n \in \mathbb{N}} \sum_{m \in M} x_{nm}^{dt} \le 1,$	$d \in D, t \in T,$	(9)
$\sum_{i \in I} \sum_{t \le 40} y_i^{dt} \le CAP,$	$d \in D$	(10)
$\sum_{i \in I} \sum_{h \in H} \sum_{t \in T} x_{ih}^{dt} = 1,$	$d \in D$	(11)
$y_i^{d41} = 0,$	$i \in I, d \in D$	(12)
$k_{ih}^{dt} \le \frac{y_i^{dt} + u_h^d}{2},$	$i\in I,\;h\in H,\;d\in D,\;t\in T$	(13)
$k_{ih}^{dt} \ge y_i^{dt} + u_h^d - 1,$	$i\in I,\;h\in H,\;d\in D,\;t\in T,$	(14)

Mathematical Model (Location Decisions)

$\sum_{h \in H} z_h = p$		(15)
$\sum_{h \in H} u_h^d = 1,$	$d \in D$	(16)
$x_{hi}^{dt} \le u_h^d,$	$i\in I,\ h\in H,\ d\in D,\ t\in T$	(17)
$x_{ih}^{dt} \le u_h^d,$	$i\in I,\ h\in H,\ d\in D,\ t\in T$	(18)
$u_h^d \le z_h,$	$h \in H, d \in D$	(19)

Mathematical Model: Scheduling Decisions

Requirements of the Problem

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2) The time intervals between the visits are fixed (services must be provided at the same slot each week)

	Freq	= 12	Frec	l= 8	Freq	= 4	Fred	2 = p	Fred	1 = 1
	DAY 1		DAY 2		DAY	DAY 3		DAY 4		5
	M-1	M-2	Tue-1	Tue-2	W-1	W-2	Th-1	Th-2	F-1	F-2
Week 1										
Week 2		_	_			_				
Week 3										
Week 4										
	3/wee	ek: conse	cutive	2 cons	/week: . or 5 apart	2/2 v cons. or	veek: 10 apart	2 cons	2/4 week: s. or 20 apa	art

Scheduling Decisions

Frequency	Doctor repeats the same tour every	
12	week	3 slots/week: consecutive
8	week	2 slots/week: either consecutive or 5 slots apart
4	two weeks	2 slots/2 weeks: either consecutive or 10 slots apart
2	_	$2 ext{ slots / 4 weeks:}$ either consecutive or $20 ext{ slots apart}$
1	_	1 slot / 4 weeks

Mathematical Model: Scheduling Decisions

Frequency of 2: 2 slots / 4 weeks: either consecutive or 20 slots apart

$y_i^{d2} + y_i^{d21} \ge y_i^{d1},$	$i \in I2, \ d \in D$	(5.20)
$y_i^{dt+1} + y_i^{dt-1} + y_i^{dt+20} \ge y_i^{dt},$	$i \in I2, \ d \in D, t \le 20: t \ne \{1, 10\}$	(5.21)
$y_i^{dt-1} + y_i^{dt+20} \ge y_i^{dt},$	$i \in I2, \ d \in D, \ t = \{10, 20\}$	(5.22)
$y_i^{dt+1} + y_i^{dt-1} + y_i^{dt-20} \ge y_i^{dt},$	$i \in I2, \ d \in D, \ 21 \le t \le 39,$	(5.23)
$y_i^{dt-1} + y_i^{dt-20} \ge y_i^{dt},$	$i \in I2, \ d \in D, \ t = \{30, 40\}$	(5.24)

Frequency of 4:

2 slots/2 weeks: either consecutive or 10 slots apart

$\sum_{t \le 20} y_i^{dt} \ge 2 \cdot u_i^d,$	$i \in I4, \ d \in D,$	(5.25)
$y_i^{d2} + y_i^{d11} \ge y_i^{d1},$	$i \in I4, \ d \in D,$	(5.26)
$y_i^{dt+1} + y_i^{dt-1} + y_i^{dt+10} \ge y_i^{dt},$	$i \in I4, \ d \in D, \ 2 \le t \le 20,$	(5.27)
$y_i^{dt+20} + y_i^{dt+30} \ge y_i^{dt} + y_i^{dt+10},$	$i \in I4, \ d \in D, \ 1 \le t \le 10,$	(5.28)
$y_i^{dt+20} + y_i^{dt+21} \ge y_i^{dt} + y_i^{dt+1},$	$i\in I4,\ d\in D, 1\leq t\leq 19:t\neq 10,$	(5.29)

Mathematical Model: Scheduling Decisions

Frequency of 8: 2 slots/week: either consecutive or 5 slots apart

$\sum_{t < 10} y_i^{dt} \ge 2 \cdot u_i^d,$	$i \in I8, \ d \in D,$	(5.30)
$y_i^{d2} + y_i^{d6} \ge y_i^{d1},$	$i \in I8, \ d \in D,$	(5.31)
$y_i^{dt+1} + y_i^{dt-1} + y_i^{dt+5} \ge y_i^{dt},$	$i \in I8, \ d \in D, \ 2 \le t \le 5,$	(5.32)
$y_i^{dt+1} + y_i^{dt-1} + y_i^{dt-5} \ge y_i^{dt},$	$i \in I8, \ d \in D, \ 6 \le t \le 10,$	(5.33)
$y_i^{dt+10} \ge y_i^{dt},$	$i \in I8, \ d \in D, 1 \le t \le 30,$	(5.34)
Frequency of 12:	3 slots/week: conse	cutive
$\sum_{i < 10} y_i^{dt} \ge 3 \cdot u_i^d,$	$i \in I12, \ d \in D,$	(5.35)
$y_i^{d2} + y_i^{d3} \ge 2 \cdot y_i^{d1},$	$i \in I12, \ d \in D,$	(5.36)
$y_i^{d1} + y_i^{d3} + y_i^{d4} \ge 2 \cdot y_i^{d2},$	$i \in I12, \ d \in D,$	(5.37)
$y_i^{dt-2} + y_i^{dt-1} + y_i^{dt+1} + y_i^{dt+2} \ge 2y_i^{dt}$	$i \in I12, \ d \in D, \ 3 \le t \le 8$	(5.38)
$y_i^{d7} + y_i^{d8} + y_i^{d10} \ge 2 \cdot y_i^{d9},$	$i \in I12, \ d \in D,$	(5.39)
$y_i^{d8} + y_i^{d9} \ge 2 \cdot y_i^{d10},$	$i \in I12, \ d \in D,$	(5.40)
$y_i^{dt+10} \ge y_i^{dt},$	$i \in I12, \ d \in D, 1 \le t \le 30,$	(5.41)

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Mathematical Model

Valid Ine	qualities	D	omain Constraints
$\sum_{i \in I12} u_i^d + \sum_{i \in I8} u_i^d \le 5,$ $\sum u_i^d \le 2$	$d \in D$	(43)	$x_{nm}^{dt}, y_i^{dt}, u_n^d, z_h, k_{ih}^{dt} \in \{0, 1\}$
$\sum_{i \in I12}^{u_i} u_i \leq 3,$ $\sum_{i \in I} u_i^d \cdot DEM_i \leq CAP,$	$a \in D$ $d \in D$	(45)	$n,m \in N, h \in H, d \in D, t \in T,$
$x_{ij}^{dt} \le u_i^d,$	$i \in I, \ j \in I, \ d \in D$	$, t \in T$ (46)	

Data: City of Burdur

Burdur data set is used for computational analysis.



Parameters

- ${\ensuremath{\, \bullet \,}}$ Coordinates \rightarrow Distances
- Population \rightarrow Frequencies
- Capacity = 40 slots
- Number of base hospitals: varied over instances

Sample Result



Sample Schedule for 3 doctors

	Mon-1	Mon-2	Tue-1	Tue-2	Wed-1	Wed-2	Thu-1	Thu-2	Fri-1	Fri-2	
Week-1	1	1	1	3	12	12	2	2	16	16	
Week-2	1	1	1	8	8	15	2	2	16	16	
Week-3	1	1	1	7	12	12	2	2	16	16	
Week-4	1	1	1	8	8	15	2	2	16	16	1
	Mon-1	Mon-2	Tue-1	Tue-2	Wed-1	Wed-2	Thu-1	Thu-2	Fri-1	Fri-2	
Week-1	25	25	25	13	13	22	11	11	5	5	
Week-2	25	25	25	21	21	20	11	11	5	5	
Week-3	25	25	25	13	13	22	11	11	5	5	
Week-4	25	25	25	21	21	20	11	11	5	5	
	Mon-1	Mon-2	Tue-1	Tue-2	Wed-1	Wed-2	Thu-1	Thu-2	Fri-1	Fri-2	
Week-1	6	6	18	18	24	24	19	19	19	17	
Week-2	10	10	23	23	24	24	19	19	19	4	
Week-3	6	6	18	18	24	24	19	19	19	26	
Week-4	10	10	9	14	24	24	19	19	19	4	



(a) Week 1 Routes





(b) Week 2 Routes



(c) Week 3 Routes

Analysis on the Problem Parameters

Effects of:

- Number of doctors
- Our Number of base hospitals
- S Frequency distribution

Number of doctors on solution times



Increasing number of doctors increase the complexity of problem, thus, solution times increase.

Number of base hospitals on solution times

Number of Doctors	mber of Number of Octors Base Hospitals		Minimum Solution Time (sec)	Average Solution Time (sec)	Maximum Solution Time (sec)	
1	1	18	17	43	180	
2	1	30	197	2,579	8,948	
2	2	24	210	2,905	5,679	
3	1	6	2,065	5798	10,933	
3	2	6	3,989	6,917	12,850	
3	3	6	3,878	8,645	18,893	

Selecting less base hospitals with the same number of doctors results in shorter solution times.

Frequency distribution on solution times

Majority of Frequency	Instance Number	Average Solution Time (sec)	-	Majority of Frequency	Instance Number	Average Solution Time (sec)
	Instance 6	2,670	-		Instance 25	29
	Instance 7	8,926		2	Instance 26	86
10	Instance 10	6,549			Instance 27	80
12	Instance 11	7,642			Instance 28	24
	Instance 14	7,283		1	Instance 20	18
	Instance 15	9,647		Ŧ	Instance 30	22
	Instance 16	3,046	-		Instance 2	200
	Instance 17	3,272			Instance 2	299
8	Instance 18	2,130			Instance J	2,002
	Instance 19	2,325		Evenly		31U 2.1E0
	Instance 20	4.597		Distributed	Instance 5	2,150
	1 1	4.470	-		Instance 8	298
	Instance 1	4,478			Instance 9	4,417
	Instance 21	4,247			Instance 12	258
4	Instance 22	2,077			Instance 13	1,186
	Instance 23	3,574				
	Instance 24	8,921				

- Dominance of frequency 1 or 2 decreases solution time.
- Majority of frequency 12 increases solution time.
- More even frequency distributions (no dominance), especially with selection of less base hospitals, result in shorter computational times.

Heuristic Algorithm

Necessity of an efficient algorithm:

- High solution times for medium/large data sets.
- Large optimality gaps at the end of time limits.

An iterative Cluster First, Route Second based approach

Construction Phase

- Determine cluster (doctor's assignments) via p-median based IP
- Route each doctor separately via adjusted PLRP model
- Add up each doctor's distance

Improvement Phase (Iterative)

- Determine next best cluster with an additional constraint to the IP
- Route each doctor separately via adjusted PLRP model
- Add up each doctor's distance
- Determine the minimum distance value among all iterations

Construction Phase

Same Parameters and Decision Variables:

set of villages I, set of base hospitals H, $DIST_{ij}$, DEM_i , CAP, p, z_h

Additional Parameters and Decision Variables:

cluster: number of clusters. i.e. number of doctors

 $x_i = \begin{cases} 1, & \text{if village } j \in I \text{ is selected as a cluster origin,} \end{cases}$

$\int (0, \text{ otherwise.})$					
$y_{ij} = egin{cases} 1, & ext{if village } i \in I ext{ is assigned to cluster origin } j \in I, \ 0, & ext{otherwise.} \end{cases}$					
	(7.1)				
	(7.2)				
$i \in I,$	(7.3)				
$j \in I$	(7.4)				
$i \in I, \ j \in I,$	(7.5)				
$j \in I,$	(7.6)				
	(7.7)				
$j \in I$	(7.8)				
$j \in I, \ h \in H,$	(7.9)				
$i,j\in I,\ h\in H,$	(7.10)				
	er origin $j \in I$, $i \in I$, $j \in I$ $i \in I, \ j \in I$, $j \in H$, $i, j \in H$, $i, j \in H$, $i, j \in H$, $i, j \in H$, $j \in $				

- $\forall j \in J, y_{ij} = 1$ are determined.
- Each *j* corresponds to a doctor.
- For each j, set I consists of i's s.t. $y_{ij} = 1$
- Index d and location decisions are removed from the PLRP model.
- For each doctor, schedules are determined via the updated model.
- Total distance is found by adding up the results of each doctor.

Improvement Phase

Find next best cluster with:

$$\sum_{i \in I} \sum_{j \in I} DIST_{ij} \cdot DEM_i \cdot y_{ij} \ge PrevIter + k$$
(7.11)

Eliminating same clusters at each iteration:

$$\sum_{s \in S} y_{sj} \le |S| - 1, \qquad S = \{i \in I : y_{i1} = 1\}$$
(7.12)

Heuristic-1: with (7.12) Heuristic-2: without (7.12)

Algorithm 1 Heuristic Approach for PLRP
Require: <i>iter</i> : Number of predetermined iterations
p-median(<i>prevIter</i>): The IP formulation explained in Chapter 7.
routing (D_j) : The IP formulation explained in Chapter 5.
1: for $i = 1$: iter do
2: if $i=1$ then
3: prevIter = 0
4: else
5: $prevIter = solution$
6: end if
7: Solve p-median $(prevIter)$
8: $solution = p-median(prevIter).objective$
9: Add new constraint (7.12)
10: $size = number of clusters$ (i.e. number of doctors)
11: Record clusters in $Doctors(size)$
12: for $j = 1$: size do
13: Solve $routing(Doctors_j)$
14: $distance_j = routing(Doctors_j).objective$
15: $j = j + 1$
16: end for
17: $Sum(i) = \sum_{j=1}^{Sum} distance_j$
18: $i = i + 1$
19: end for
20: $Result = \min_{i=1iter} Sum(i)$

24 instances of small data set with 20 iterations

Results of Heuristics

	Mathematical Model			Heuristic-1			Heuristic-2			
	Obj. Value	Solution Time (sec)	Obj. Value	ltera- tion	Solution Time (sec)	Gap (%)	Obj. Value	ltera- tion	Solution Time (sec)	Gap (%)
lns 1	5,281.18	6,684	5,281.18	4	179	0.00%	5,281.18	7	219	0.00%
Ins 2	4,943.81	357	4,943.81	1	106	0.00%	4,943.81	1	124	0.00%
Ins 3	4,491.33	1,180	4,491.33	1	104	0.00%	4,491.33	1	122	0.00%
Ins 4	4,805.67	344	4,805.67	1	95	0.00%	4,805.67	1	130	0.00%
Ins 5	4,876.04	2,677	4,876.04	2	140	0.00%	4,876.04	2	173	0.00%
Ins 6	7,885.68	2,989	7,908.40	2	94	0.29%	7,908.40	2	119	0.29%
Ins 7	7,908.40	6,773	7,908.40	1	93	0.00%	7,908.40	1	106	0.00%
Ins 8	6,053.13	303	6,053.13	1	120	0.00%	6,053.13	1	121	0.00%
Ins 9	4,856.33	5,483	4,856.33	1	122	0.00%	4,856.33	1	118	0.00%
lns 10	6,097.48	5,926	6,097.48	2	80	0.00%	6,097.48	3	116	0.00%
lns 11	6,012.08	12,742	6,012.08	2	78	0.00%	6,012.08	3	119	0.00%
lns 12	5,551.66	197	5,551.66	14	102	0.00%	5,635.70	1	124	1.51%
lns 13	4,235.74	210	4,235.74	1	108	0.00%	4,235.74	1	126	0.00%
Ins 14	5,594.12	3,989	5,594.12	16	78	0.00%	5,978.96	1	115	6.88%
lns 15	5,179.78	3,878	5,179.78	5	80	0.00%	5,196.60	17	99	0.32%
lns 16	5,310.06	3,259	5,447.26	1	166	2.58%	5,447.26	1	195	2.58%
lns 17	4,863.60	3,381	4,926.29	1	158	1.29%	4,926.29	1	214	1.29%
lns 18	5,002.72	2,569	5,002.72	3	148	0.00%	5,002.72	4	206	0.00%
lns 19	4,663.24	3,669	4,724.97	3	152	1.32%	4,663.24	10	195	0.00%
Ins 20	5,310.06	2,890	5,310.06	6	173	0.00%	5,310.06	14	203	0.00%
lns 21	4,895.42	5,071	4,895.42	1	153	0.00%	4,895.42	1	193	0.00%
Ins 22	3,059.73	1,494	3,353.17	2	198	9.59%	3,287.80	6	225	7.45%
Ins 23	2,601.60	1,127	2,601.60	1	214	0.00%	2,601.60	1	228	0.00%
Ins 24	2,984.28	6,978	3,103.14	17	242	3.98%	3,179.24	17	315	6.53%
		3507			132	18 opt 0.79%			162	16 opt 1.12%

Conclusions

Conclusions:

- An IP for PLRP is developed which determines the schedules via its constraints, satisfies certain visiting alternatives, dedicates each doctor to the villages and selects base hospitals.
- Output and the second studies indicated small instances can be solved in reasonable times; however, this is not valid for medium and large ones.
 - Higher number of doctors result in higher solution times.
 - The lower number of base hospitals to select, the less solution times.
 - Even frequency distributions shorten the computational times.
- Iterative heuristic methodology based on a "Cluster First, Route Second" approach determines optimal or near-optimal solutions in shorter times.
 - Both variants have their own disadvantages.
 - Heuristic-1 always provides solutions in shorter times.
 - Heuristic-1 provides lower distance values in the majority of the cases.

Other applications

- Specialists services
- Follow up patients
- Healthcare services for refugee camps!
- COVID-19 test booths....