The Demand- Selective Location Routing Problem: the School Districting Application

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School Districting

- Eight years primary education is obligatory
- In some districts, there aren't any primary school
- School Districting Program
- Students in these districts are transported to districts with primary schools
- Central Schools

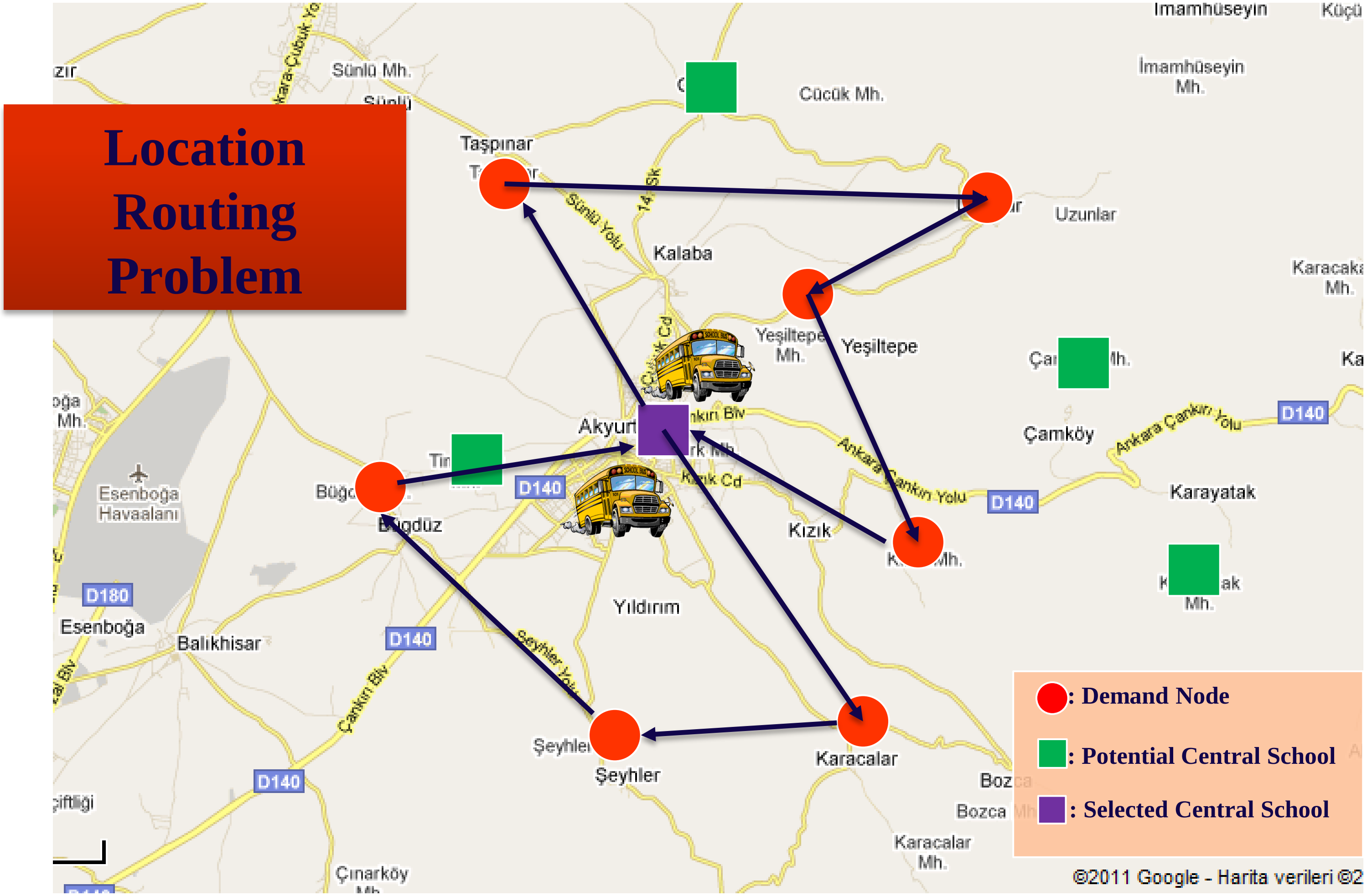
School Districting

- 1989-90 → 5 Central Schools and 305 students.
- 2009-10 → 80 cities with 667,475 students
- Now → 5,754 central schools serving students from 39,559 districts in Turkey
- Mathematical models are developed
- Heuristic approaches for minimizing the cost of school districting application
- **Real Life Scenarios**

School Districting

- School Districting
  - Finding the bus routes to carry the children to central schools
  - Selecting the central school(s)
    - Building a central school is also a possibility but it is too costly
    - Therefore, the central school is selected from one of the currently operating schools
    - Number of Central Schools
      - Geographical distribution of demand nodes
      - The capacity of the central schools
      - Number of students in the demand nodes

School Districting



School Districting

Location Routing Problem

- the selection of central school location
- assignment of demand sites to central school
- transportation routes between demand nodes

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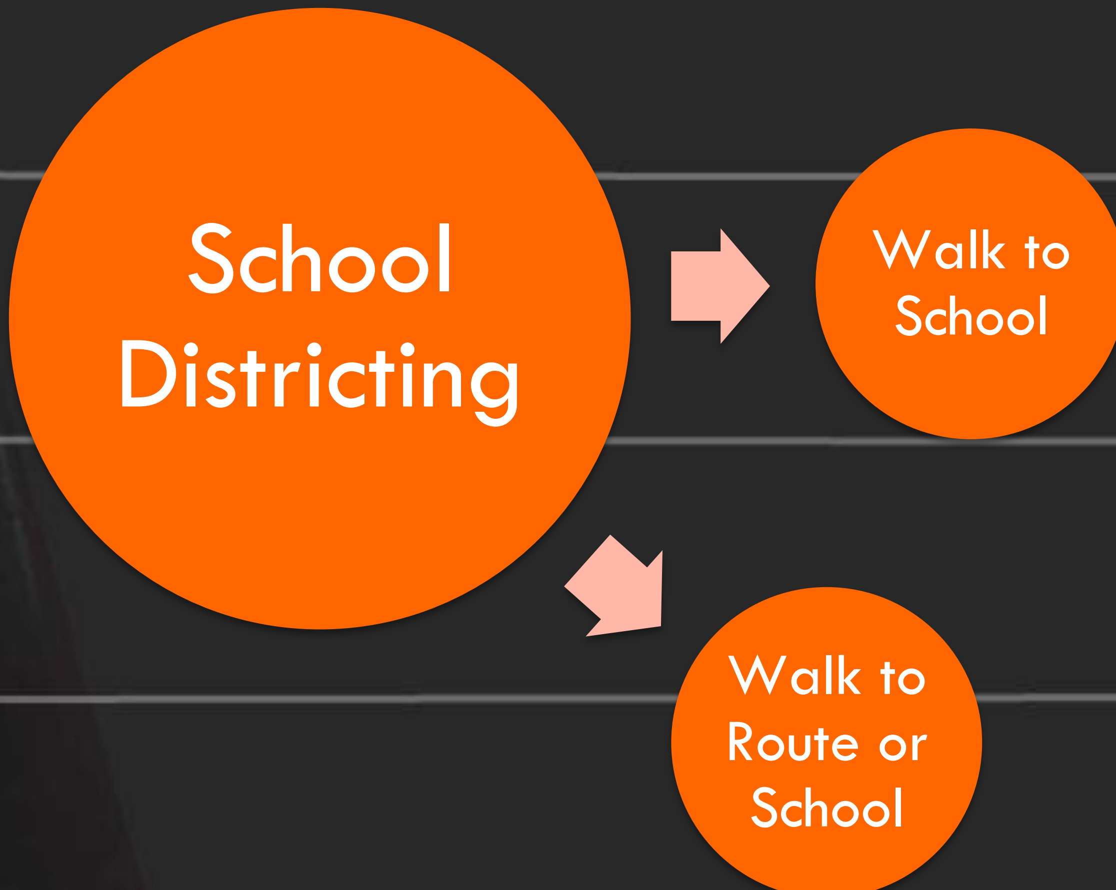
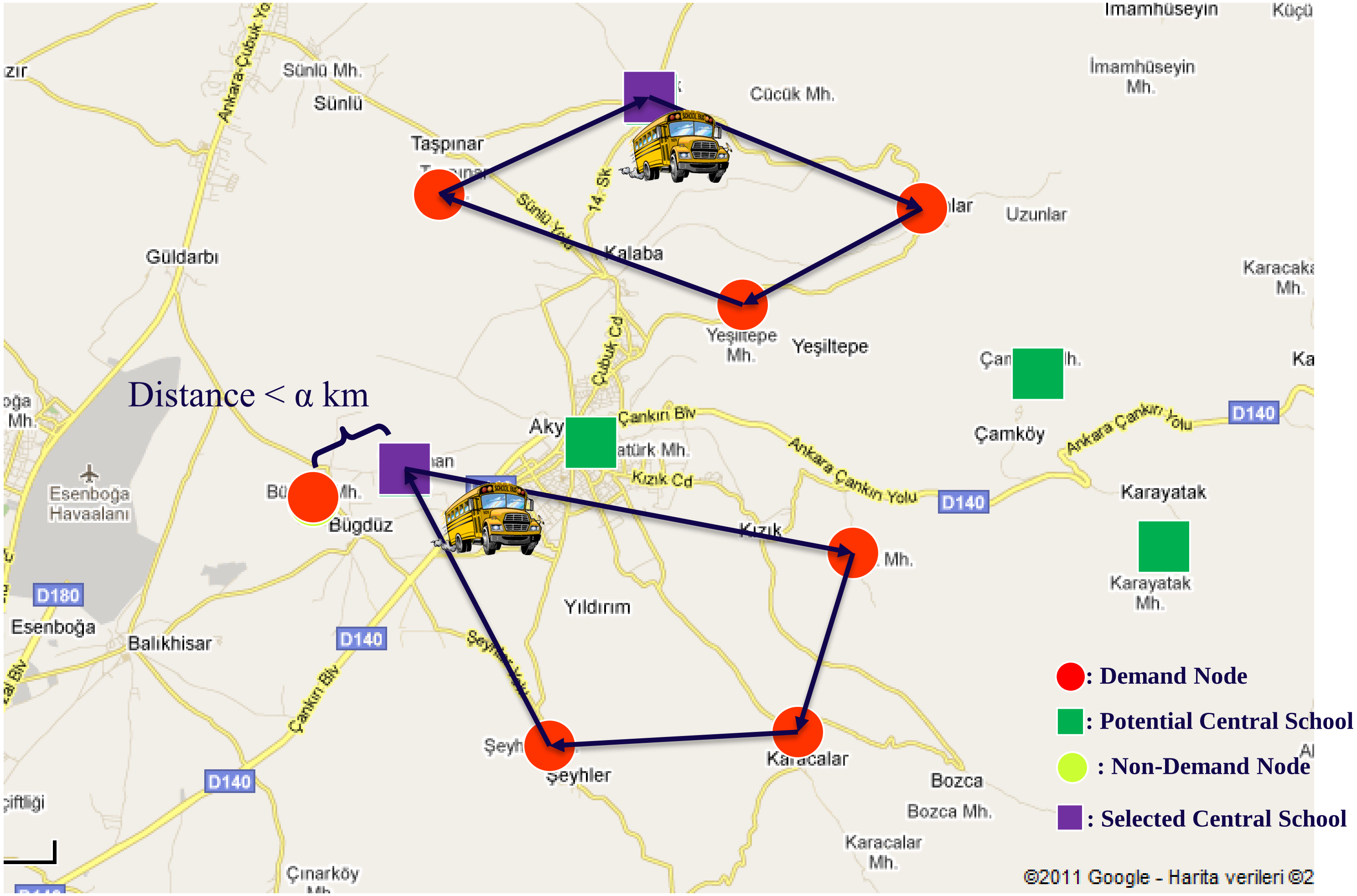
Demand Selective Location Routing Problem

the students at a demand site may walk directly to central school or to other demand sites to be picked up.

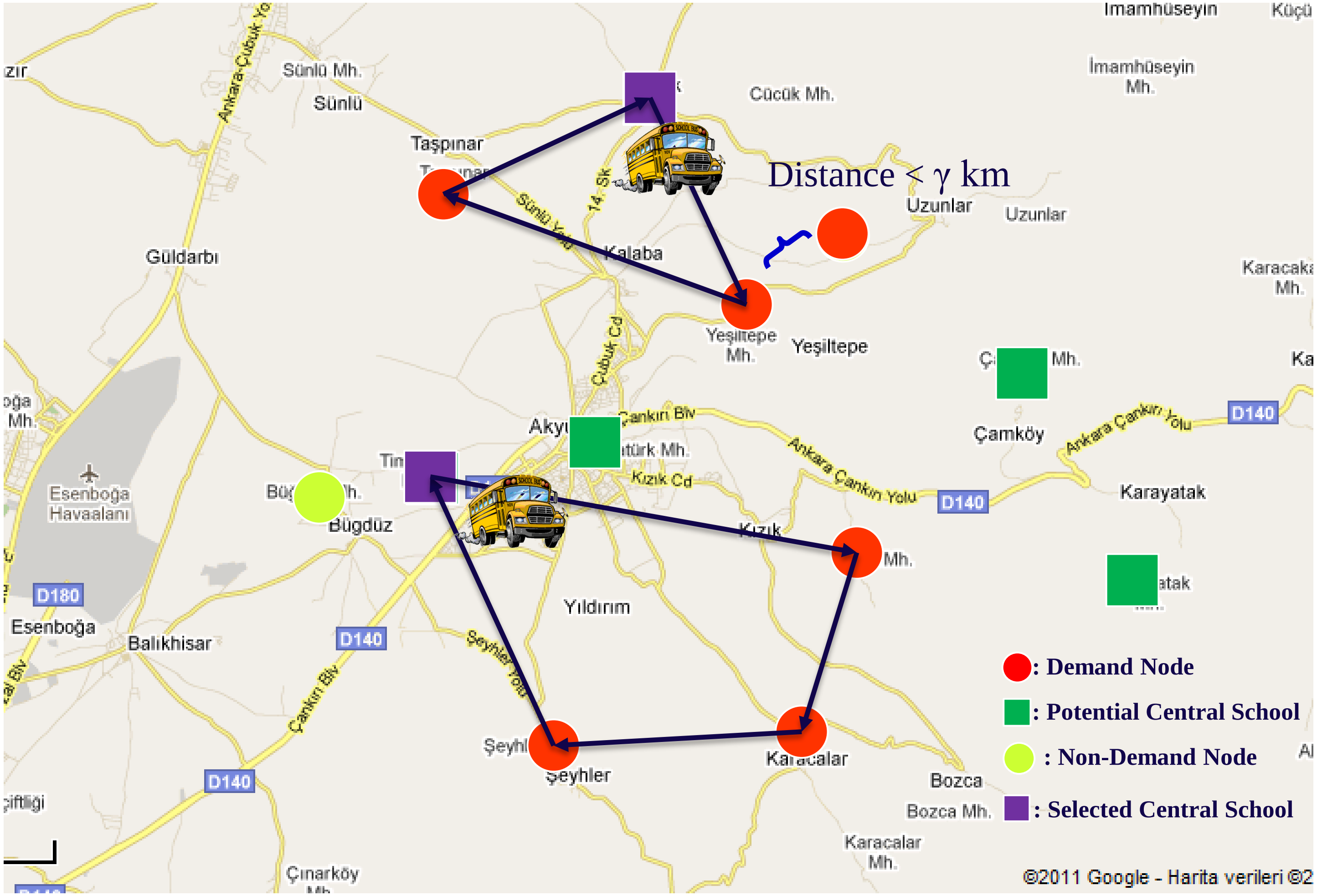
School Districting



School Districting | Walk to School



School Districting | Walk to Route or School



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Mathematical Model | Walk To School

$$s_i = \begin{cases} 1, & \text{if demand node } i \in D \text{ needs to be visited by a vehicle} \\ 0, & \text{otherwise} \end{cases}$$

$$x_j = \begin{cases} 1, & \text{if candidate location } j \in C \text{ is selected as a central school location} \\ 0, & \text{otherwise} \end{cases}$$

$$z_{ijk} = \begin{cases} 1, & \text{if node } i \text{ immediately precedes node } j \text{ using bus of } k^{th} \text{central school; } i,j \in N, k \in C \\ 0, & \text{otherwise} \end{cases}$$

y_{ijk}: Number of students on a bus of k^{th} *central school, exiting from node i right before node j;*

$i \in D; j, k \in C$

SD_j : Set of all demand nodes that the distance to j $\in$ D is less than or equal to α km

$SD_j = \{ \text{i: } I_{ij} \leq \alpha \text{ , i} \in \text{D} \}$

SC_i : Set of all candidate central school nodes that the distance to node i $\in$ N less than or equal to α km, $SC_i = \{ \text{j: } I_{ij} \leq \alpha \text{ , j} \in \text{C} \}$.

$SR_i = \{ \text{h:h} \in \text{D, } \; l_{ih} \leq \gamma \}$.

Model

$$\text{Min} \sum_{i \in N} \sum_{j \in N} \sum_{k \in C} c_{ij} z_{ijk} \quad (1)$$

s.t.

$$\sum_{k \in C} \sum_{j \in N} z_{ijk} = s_i \qquad \forall i \in D \quad (2)$$

$$1 - x_j \geq s_i \qquad \forall i \in SD_j, \forall j \in C \quad (3)$$

$$1 - \sum_{j \in SC_i} (x_j) \leq s_i \qquad \forall i \in D \quad (4)$$

$$\sum_{j \in C} x_j = p \quad (5)$$

Model

$$\sum_{j\in N}\sum_{k\in C} z_{jik} = \sum_{j\in N}\sum_{k\in C} z_{ijk} \qquad \forall j \in D \qquad (6)$$

$$\sum_{i\in D} z_{ijj} = \sum_{i\in D} z_{jij} \qquad \forall j \in C \qquad (7)$$

$$z_{ijk} \leq x_k \qquad \forall i,j \in N, \forall k \in C \qquad (8)$$

$$y_{ijk} \geq z_{ijk} \qquad \forall i,j \in N, \forall k \in C \qquad (9)$$

$$y_{ijj} = z_{jij} \qquad \forall i \in D, \forall j \in C \qquad (10)$$

$$\sum_{j\in N} y_{ijk} = \sum_{r\in N} y_{rik} + \sum_{j\in N} q_i z_{ijk} \qquad \forall i \in D, \forall k \in C \qquad (11)$$

$$y_{ijk} \leq QV z_{ijk} \qquad \forall i,j \in N, \forall k \in C \qquad (12)$$

$$\sum_{i\in D} y_{ikk} \leq QS \qquad \forall k \in C \qquad (13)$$

$$x_j, s_i, z_{ijk} \in 0,1;\ y_{ijk} \geq 0, integer \qquad \forall i \in D, \forall j,k \in C \qquad (14)$$

EXTENSIONS

Model | Walk to Route or School

District must be visited if and only if there is no central school within α km, and no demand nodes which is visited by a node within γ km.

Min (1)

s.t.

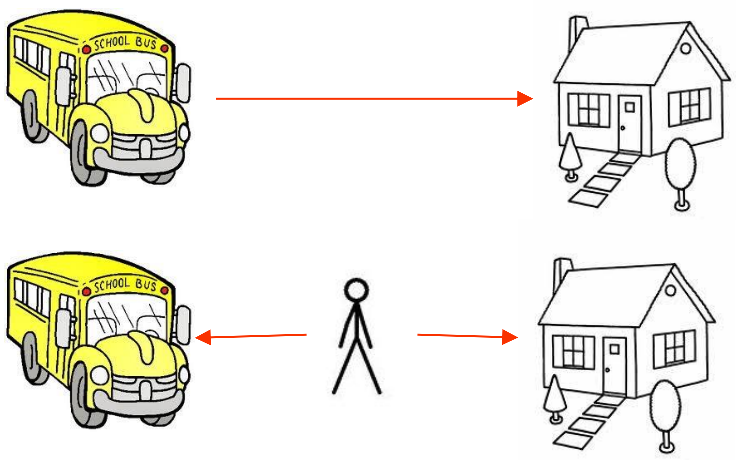
(2) , (3), (5) - (14)

$$1 - \sum_{j \in Candidate_i} (x_j) \leq s_i \qquad \forall i \in I$$

$$1 - \sum_{j \in SC_h} x_j - \sum_{m \in N} \sum_{k \in j} \sum_{i \in SR_h} z_{mik} \leq s_h \qquad \forall h \in D \qquad (4')$$

Model Extension| Distance Constrained Walk to Route or School

- Students
  - Central School
  - + Another District
  - **+ Travel Distance Constraint**



Model Extension| Distance Constrained Walk to Route or School

Min (1)

s.t.

(2) , (3), (4'), (5) - (14)

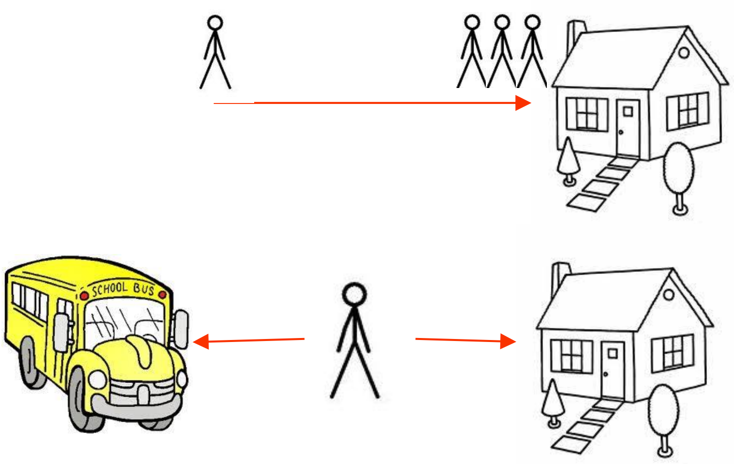
$$t_{ijk} \leq B\, z_{ijk} \qquad \forall i, j \in N, \forall k \in C \qquad (15)$$

$$\sum_{j \in N} t_{ijk} = \sum_{r \in N} t_{rik} + \sum_{j \in N} l_{ij}\, z_{ijk} \qquad \forall i \in D, \forall k \in C \qquad (16)$$

$$t_{ijk} \geq 0 \qquad \forall i, j \in N, \forall k \in C \qquad (17)$$

Model Extension| Cumulative Walk to Route or School

- Students
  - Central School
  - + Another District
  - + Travel Distance Constraint
  - **The objective is not related to buses, but to the number of students**



Model Extension| Cumulative Walk to Route or School

Model Extension| Cumulative Walk to Route or School

$$\text{Min} \sum_{i \in N} \sum_{j \in N} \sum_{k \in C} c'_{ij}\, y_{ijk} \qquad (1')$$

s.t.

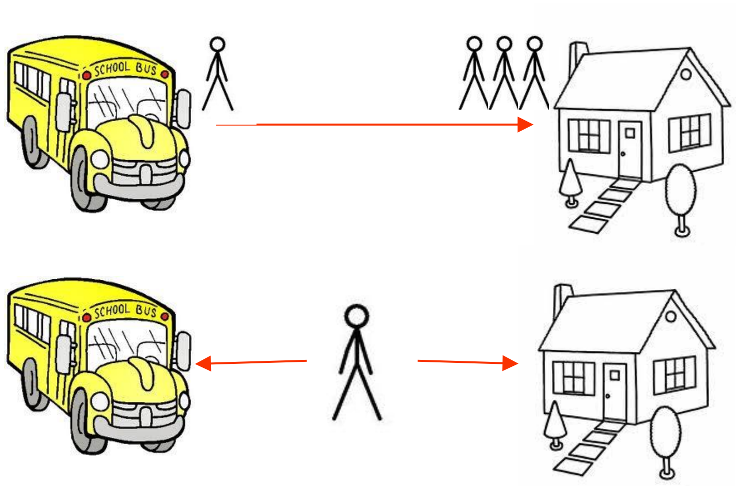
(2) , (3), (4'), (5) – (14)

$$\sum_{i \in D} z_{jij} \leq R\, x_j \qquad \forall j \in C \qquad (18)$$

$$\sum_{i \in D} z_{ijj} \leq R\, x_j \qquad \forall j \in C \qquad (19)$$

Model Extension| D-Cum WTRS

- Students
  - Central School
  - + Another District
  - + Travel Distance Constraint
  - The objective is not related to buses, but to the number of students



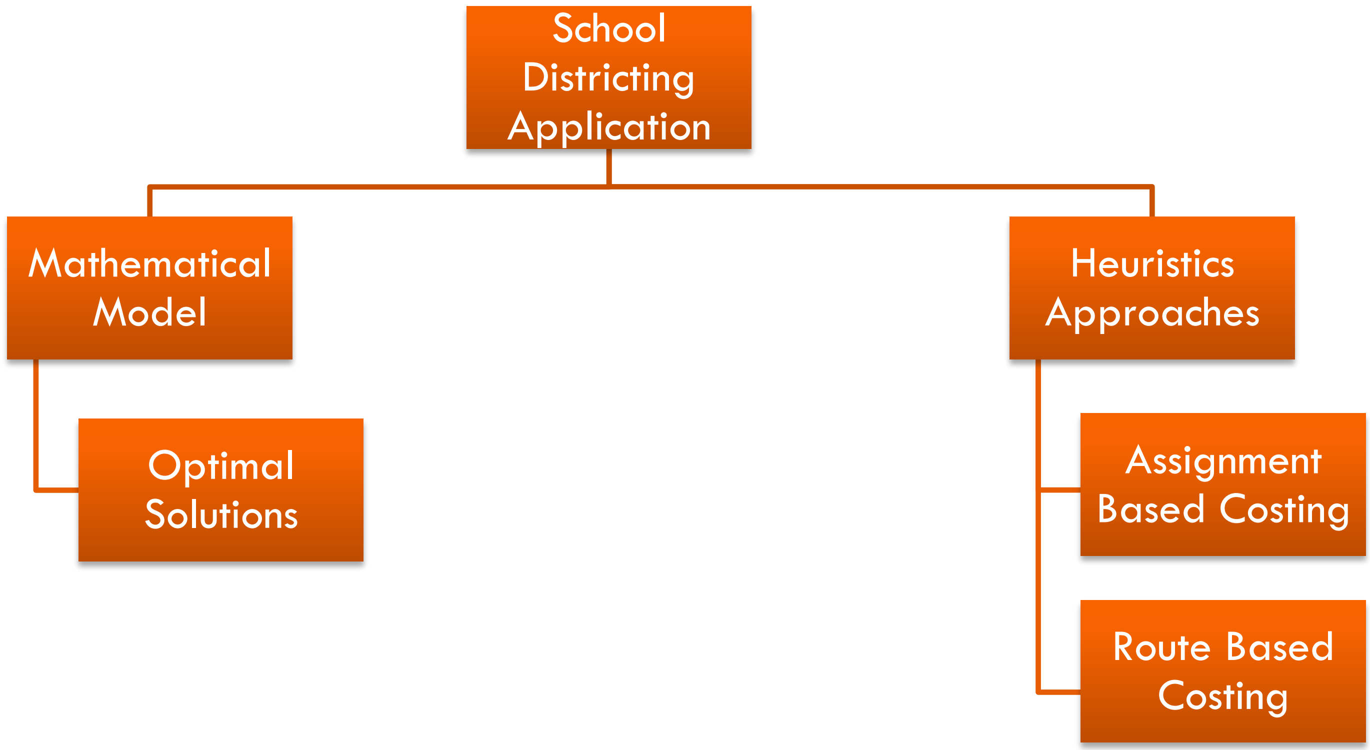
Model Extension| D-Cum WTRS

$$\text{Min} \quad (1')$$

s.t.

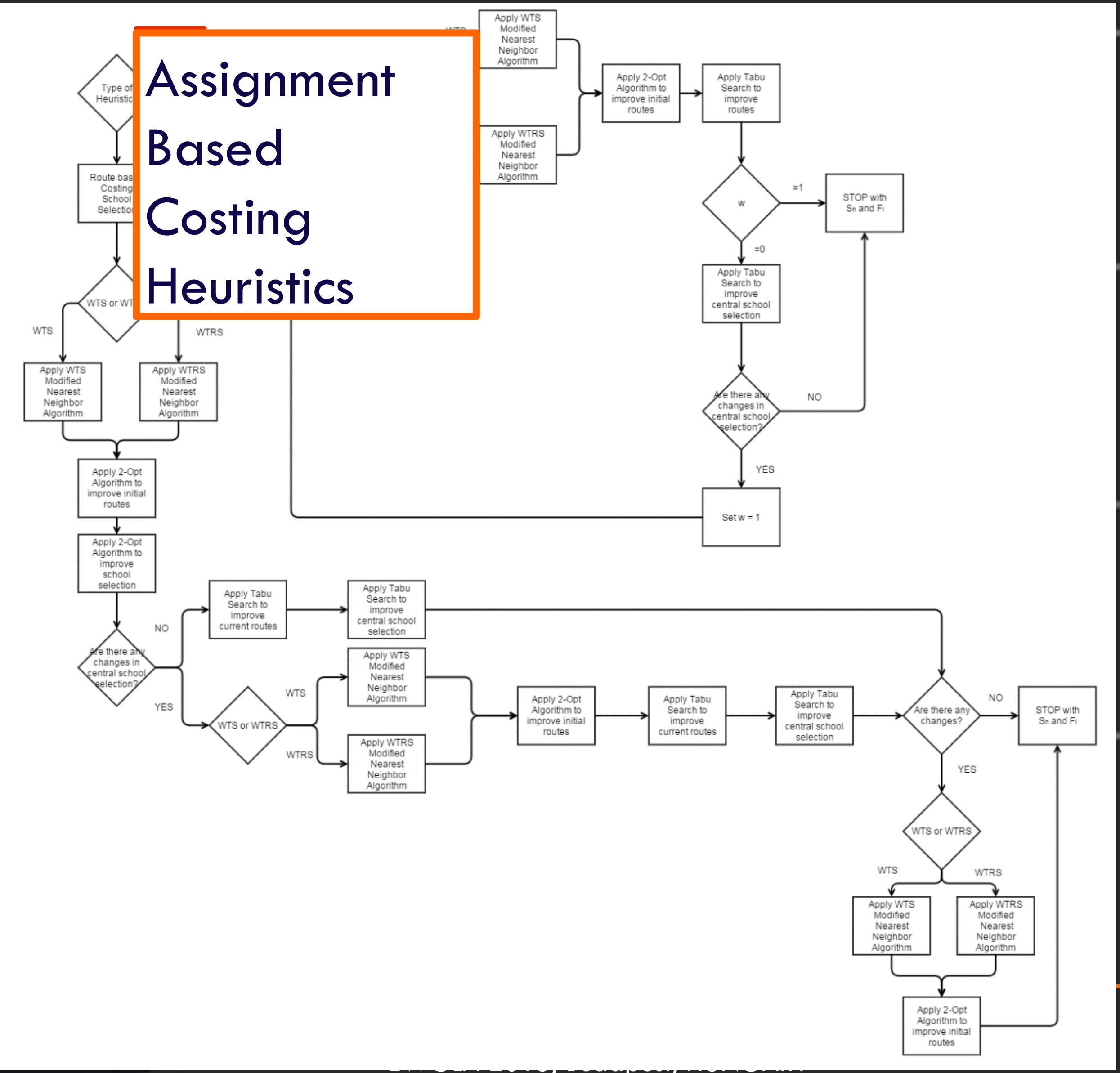
$$(2),(3),(4'),(5)-(19)$$

Solution Methodologies



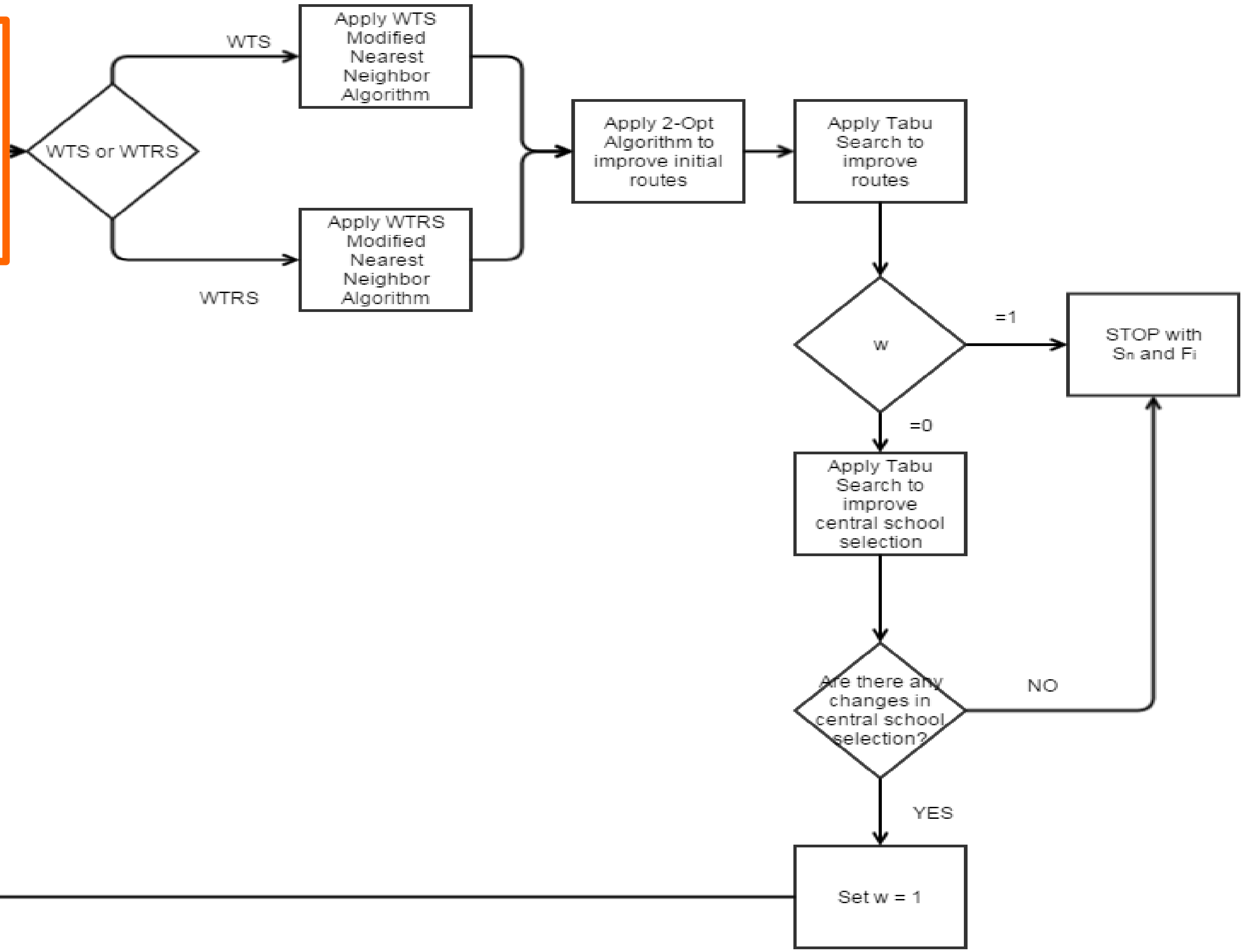
Heuristics

Assignment Based Costing Heuristics



Heuristics

Incremental
Greedy
Sequential
School Selection



Heuristics | Incremental Greedy Sequential School Selection

D: Set of distircts

PS: Set of possible central schools

PSC: Copy set of possible central schools

S^n: Set of Selected Central Schools

CS^P = {all P-tuple of i ϵ PSC}

Q^P : Set of cost for CS^P

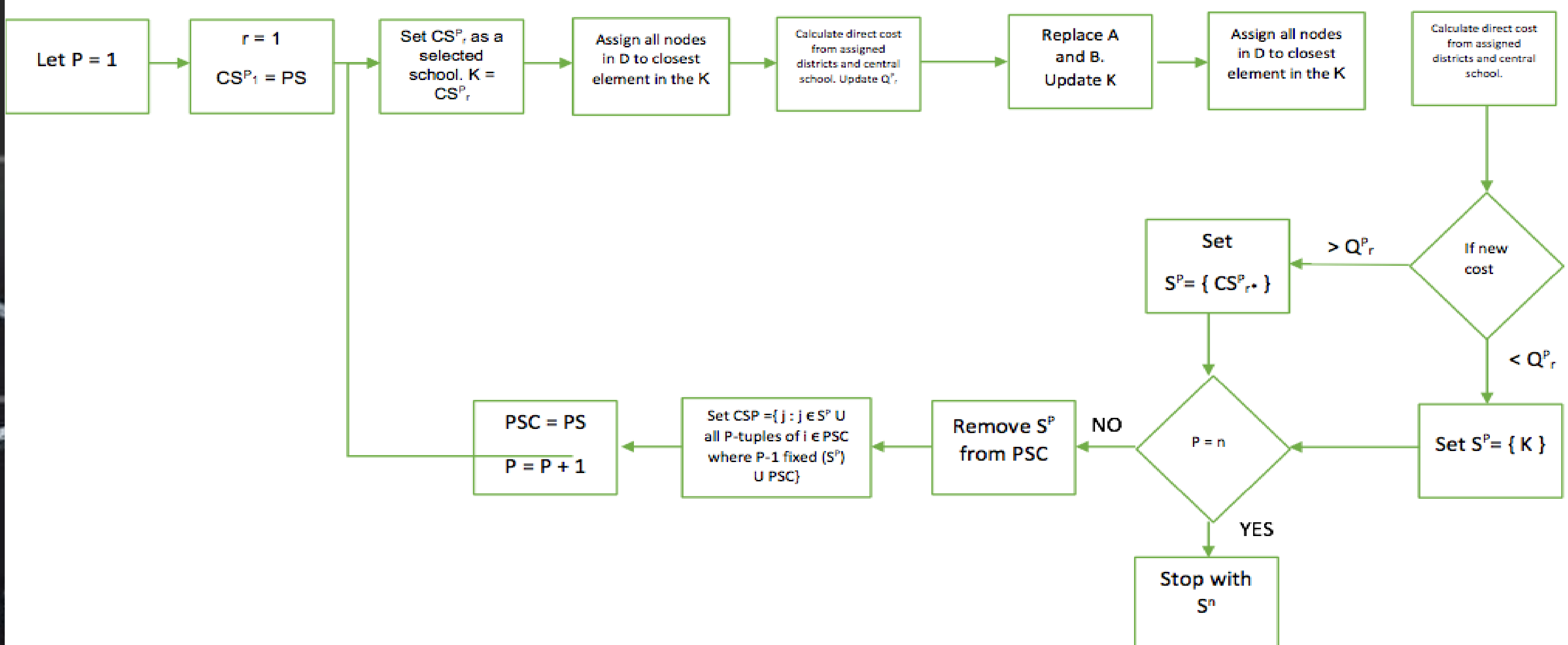
CS^P_r represents the r^{th} P-tuple of CS^P

Q^P_r reprensents the r^{th} element of Q^P

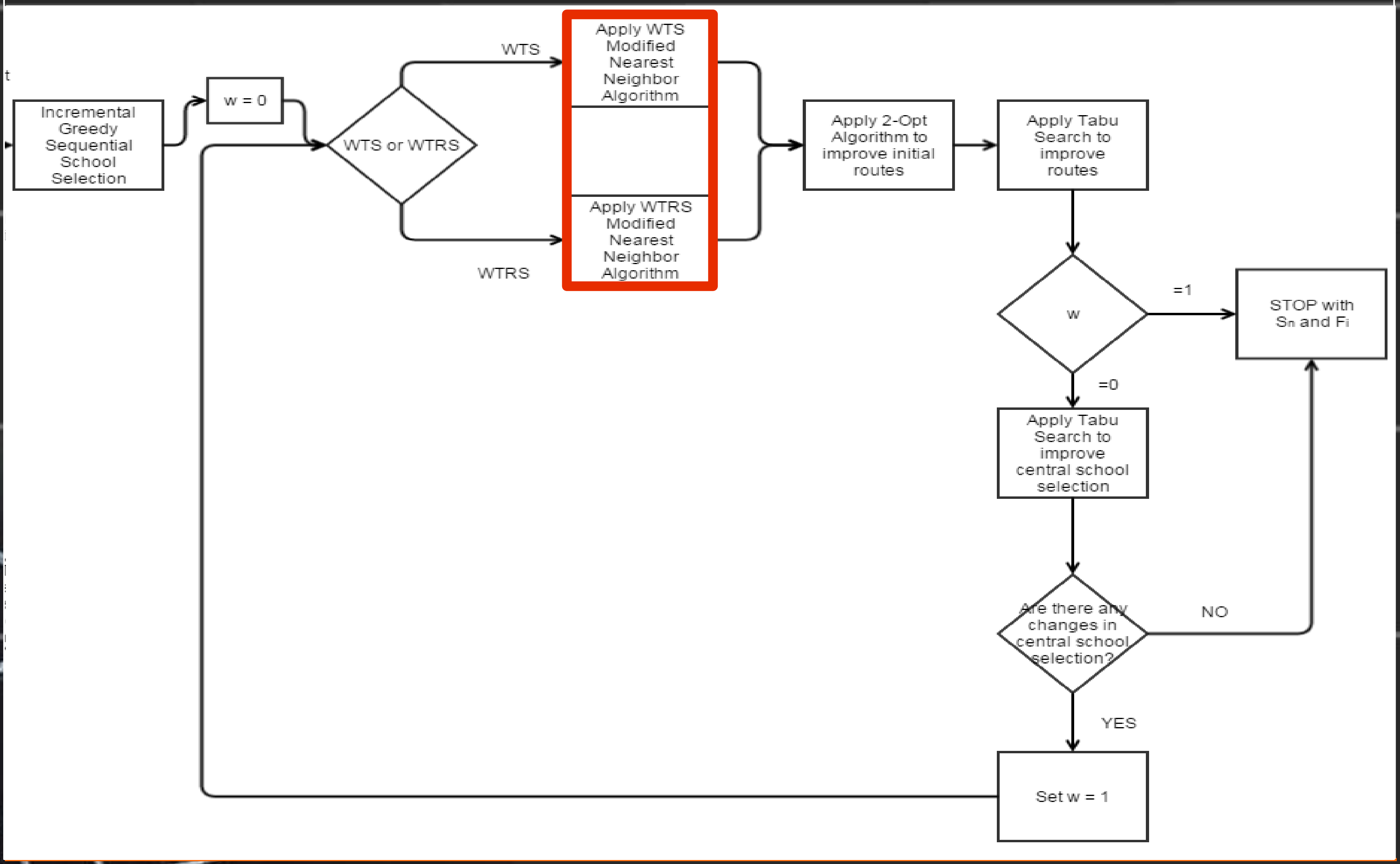
$|CS^P| = R$

A: Set of central schools in CS^P_r

B: Set of central schools not in CS^P_r



Heuristics


Incremental Greedy Sequential School Selection
w = 0
WTS or WTRS
WTS
WTRS
Apply WTS Modified Nearest Neighbor Algorithm
Apply WTRS Modified Nearest Neighbor Algorithm
Apply 2-Opt Algorithm to improve initial routes
Apply Tabu Search to improve routes
w
=1
=0
STOP with Sn and Fi
Apply Tabu Search to improve central school selection
Are there any changes in central school selection?
NO
YES
Set w = 1

Heuristics | Nearest Neighbour Algorithm

W_i: Set of WTS

WR_i: Set of WTRS

R_i: Set of districts which are assigned to be picked up by school bus

F_i: Set of final routes

I_{ij}: Set of distances between ith school and jth district

K_{mj}: Set of distances between ith district and jth district

w: tabu search decision variable

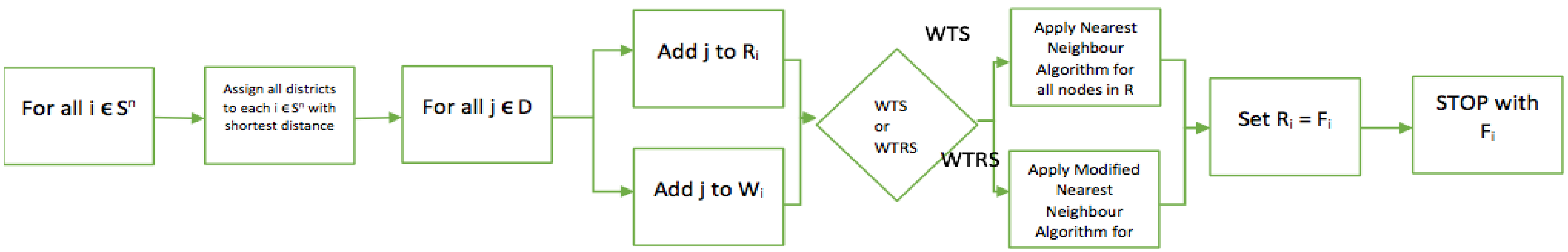
W_1={}

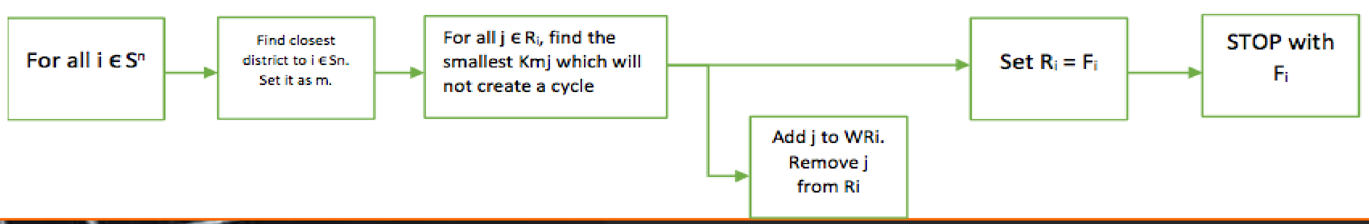
WR_1={}

R_1={}

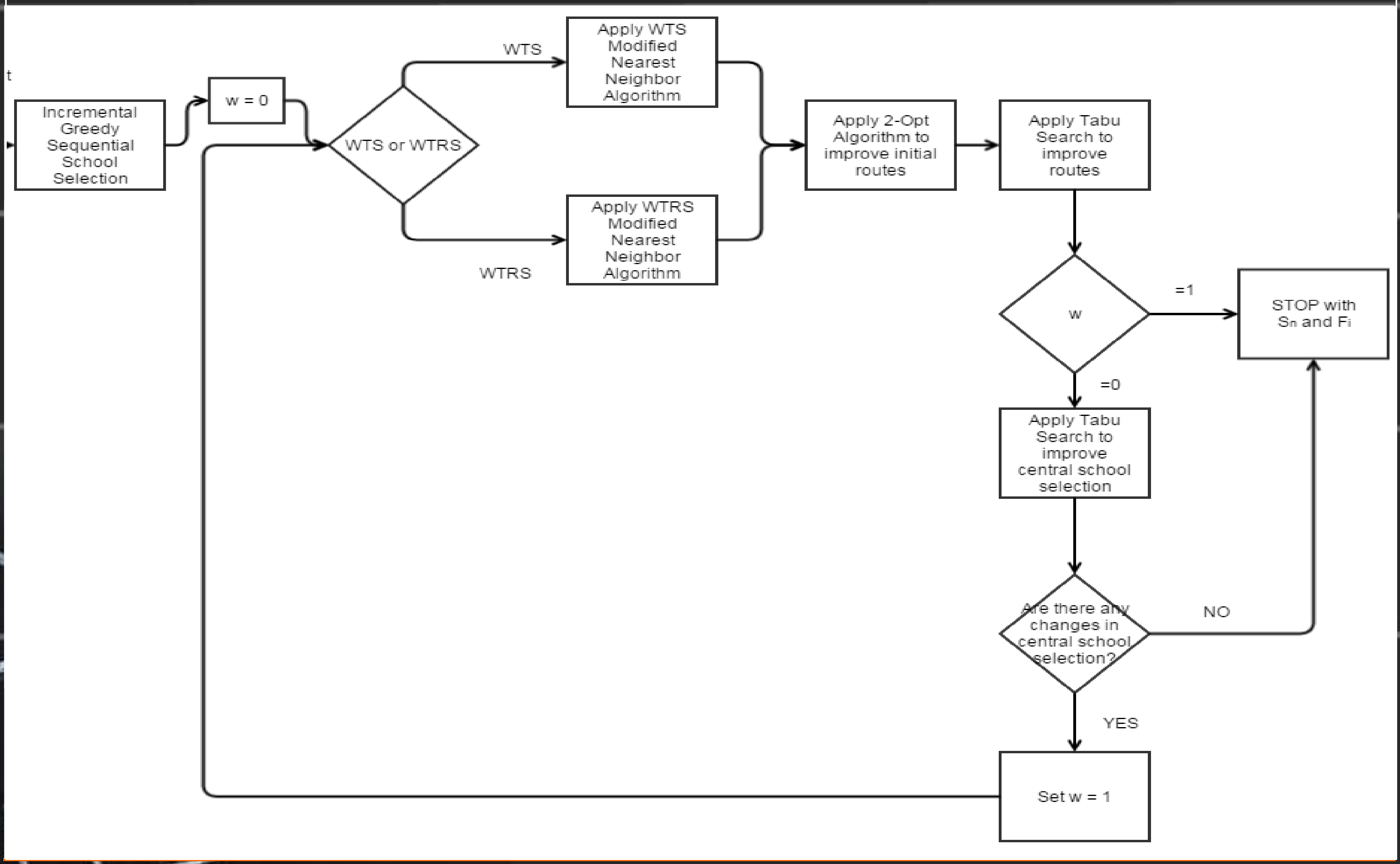
F_1={}

w = 0





Heuristics



Incremental Greedy Sequential School Selection
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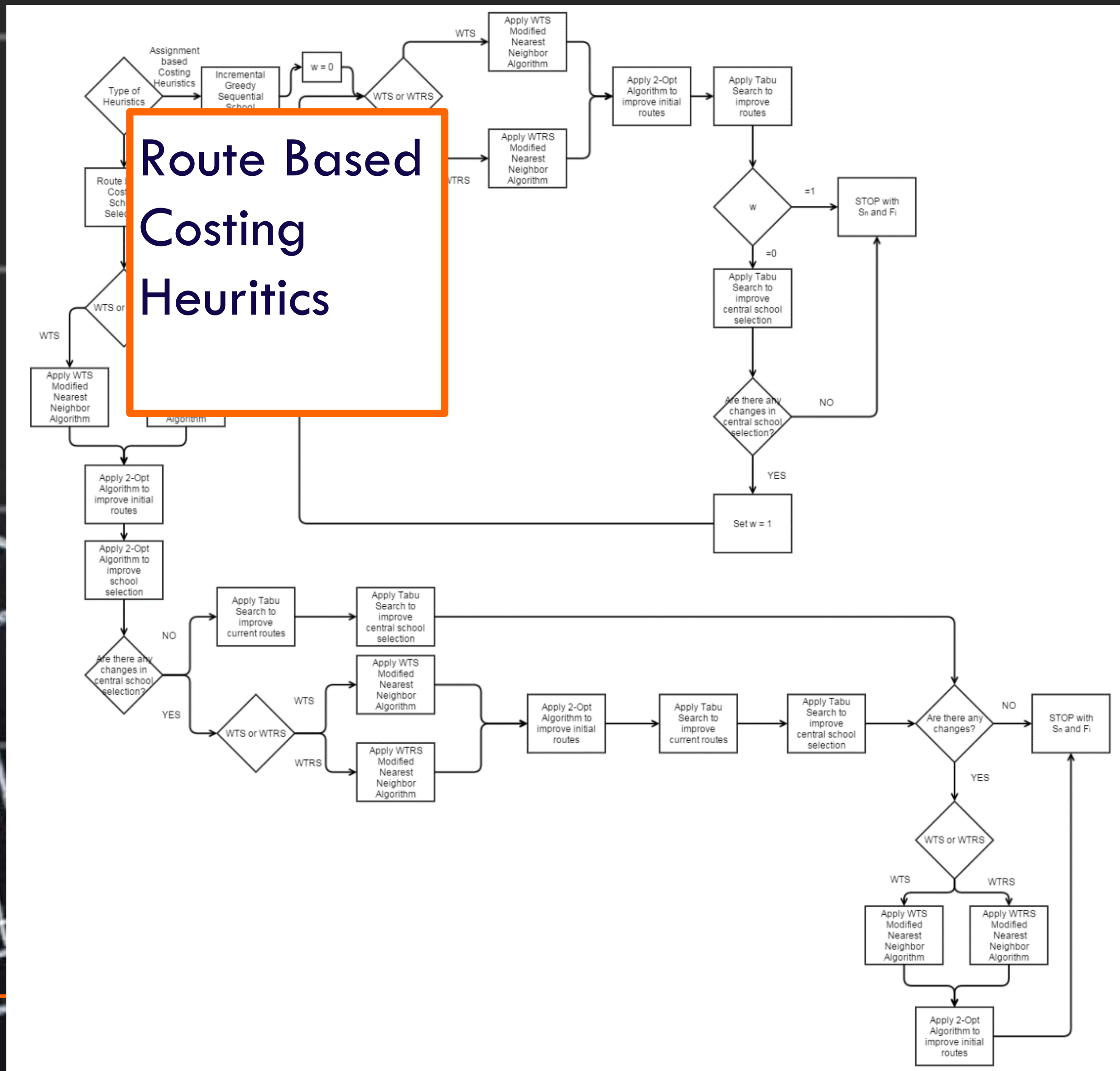
NO
YES
Set w = 1

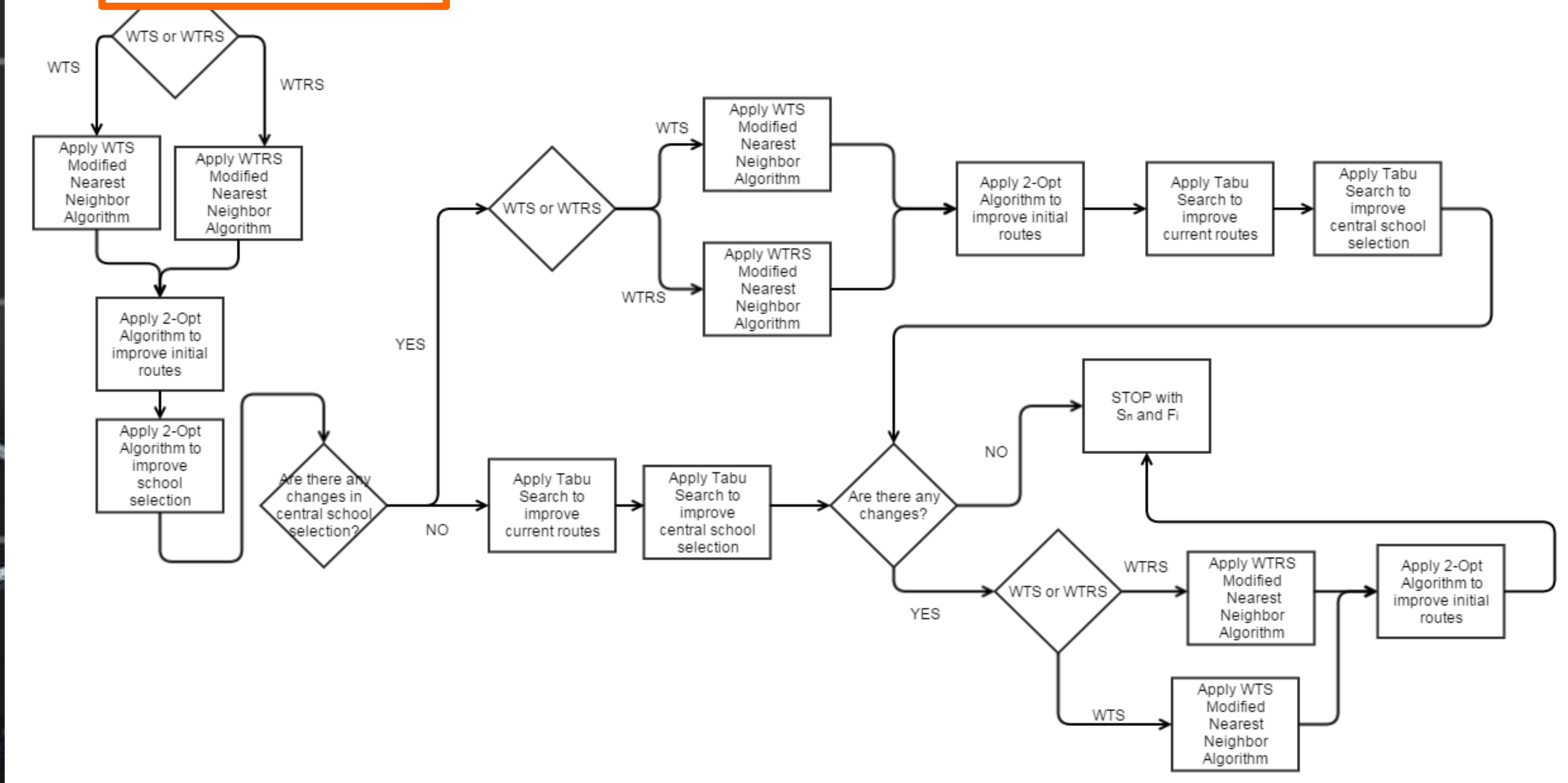
Heuristics

Route Based Costing Heuritics



Heuristics

Route
Based Costing
School Selection



Heuristics | Route Based Costing School Selection

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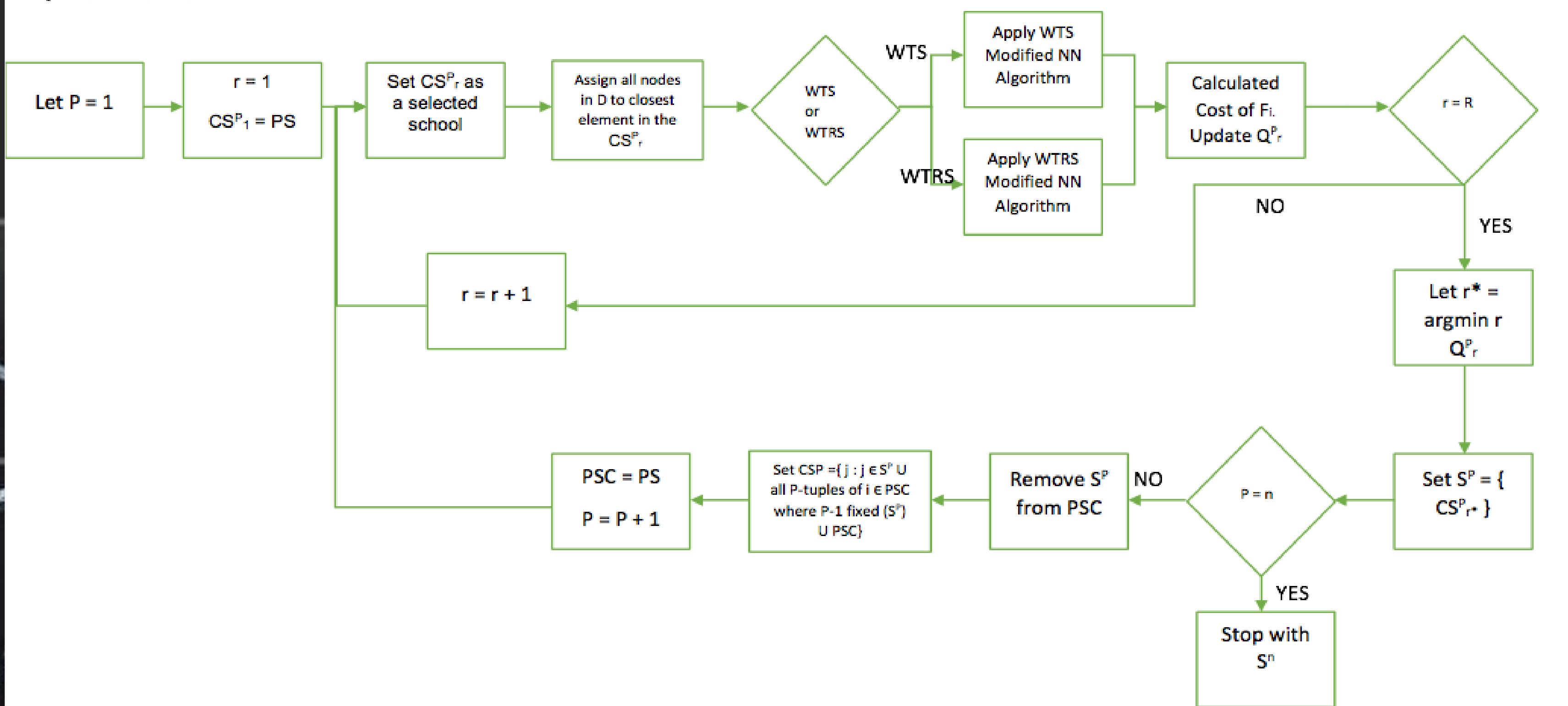
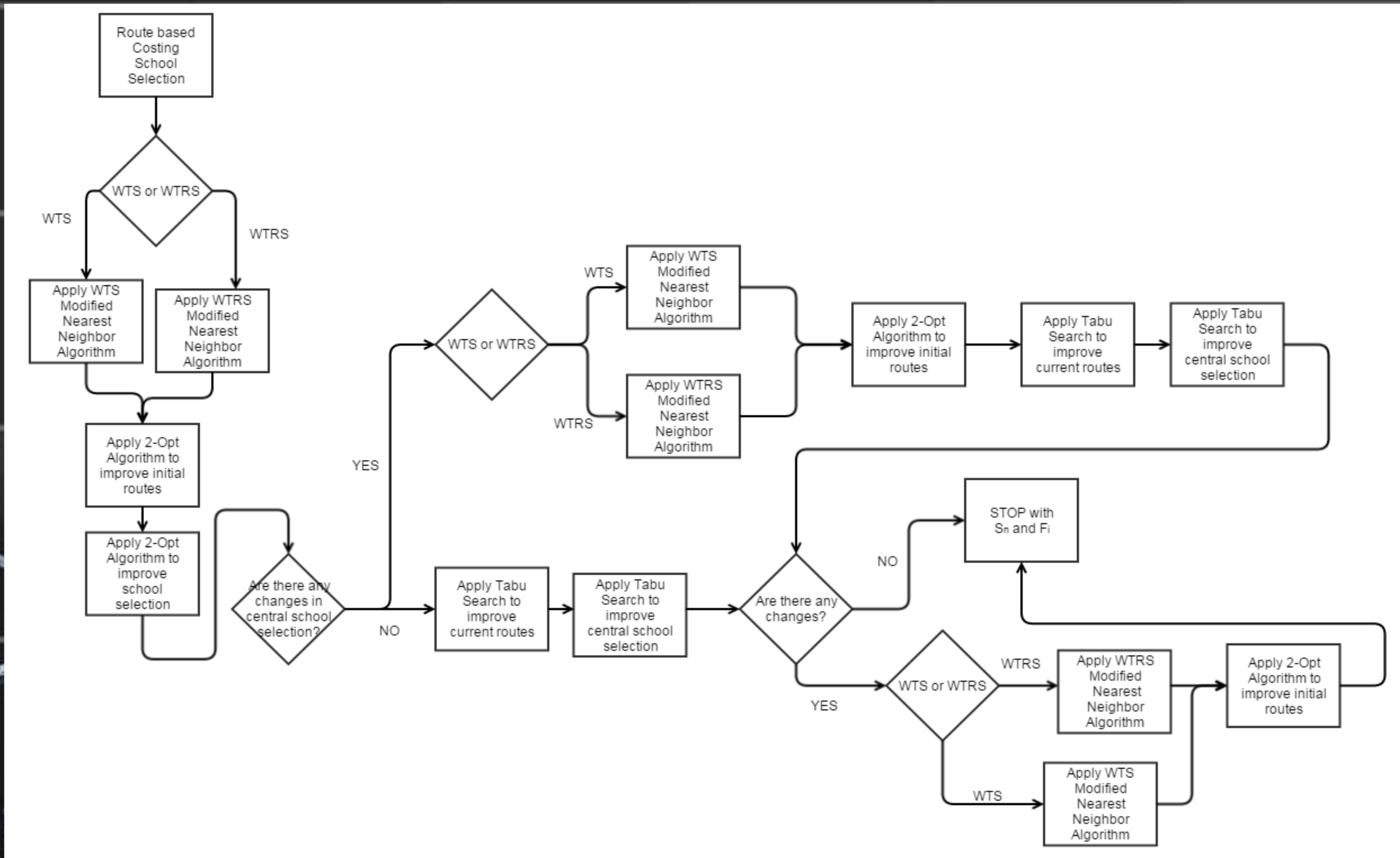
Q^P_r reprensents the r^{th} element of Q^P

$|CS^P| = R$

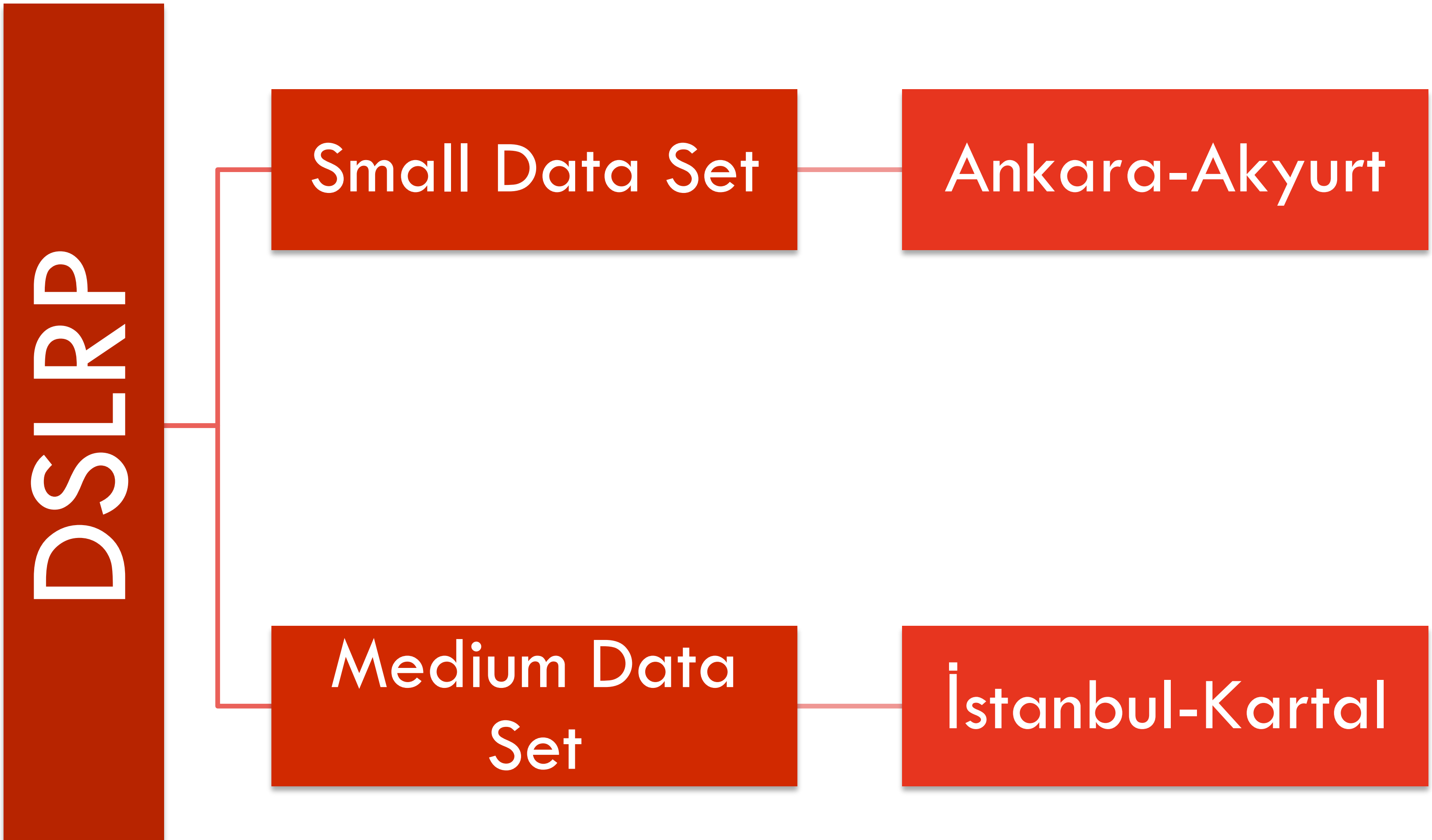


Heuristics

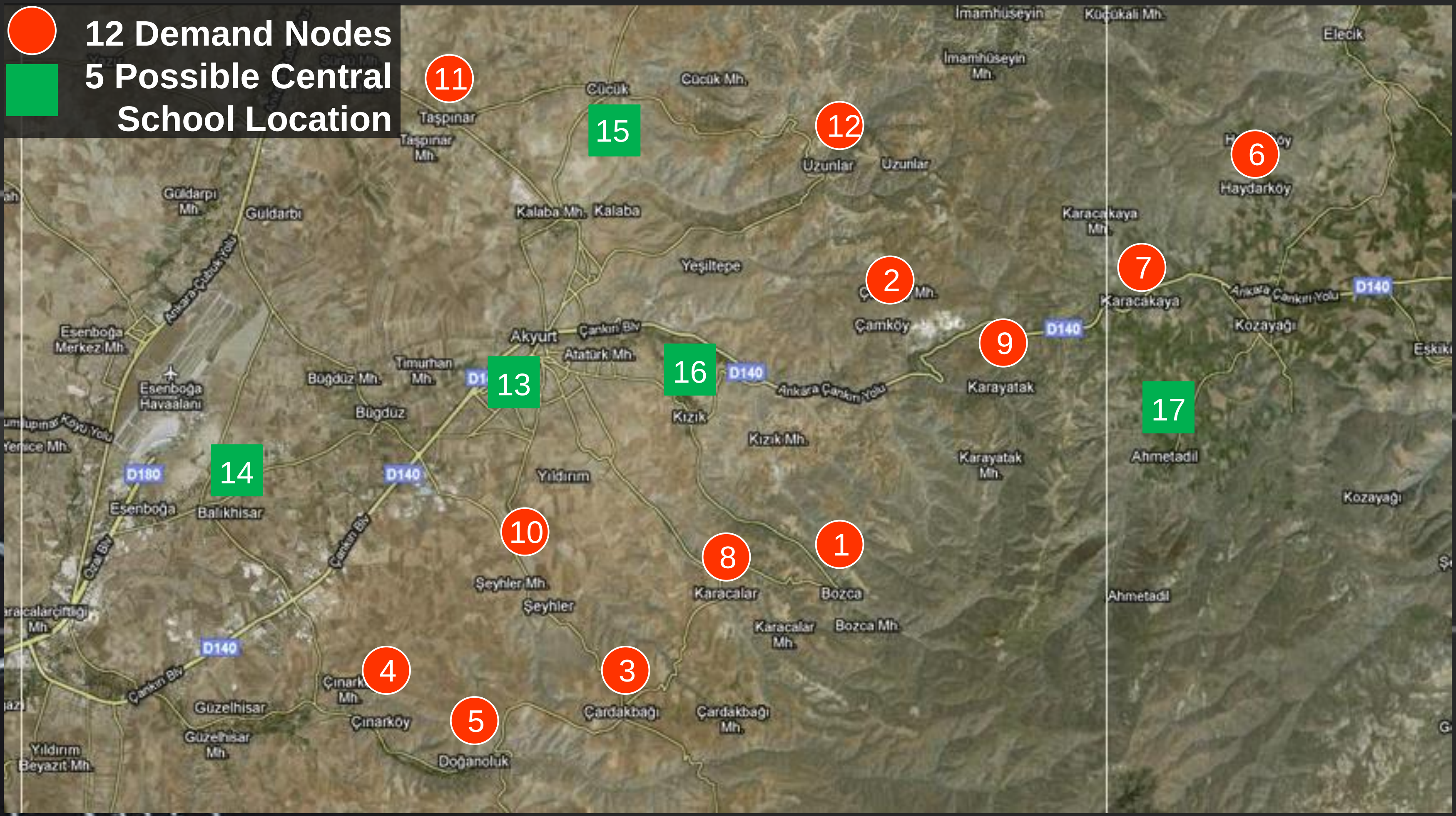


Route based Costing School Selection
WTS or WTRS
WTS
WTRS
Apply WTS Modified Nearest Neighbor Algorithm
Apply WTRS Modified Nearest Neighbor Algorithm
Apply 2-Opt Algorithm to improve initial routes
Apply 2-Opt Algorithm to improve school selection
Are there any changes in central school selection?
YES
NO
WTS or WTRS
WTS
WTRS
Apply WTS Modified Nearest Neighbor Algorithm
Apply WTRS Modified Nearest Neighbor Algorithm
Apply 2-Opt Algorithm to improve initial routes
Apply Tabu Search to improve current routes
Apply Tabu Search to improve central school selection
Apply Tabu Search to improve current routes
Apply Tabu Search to improve central school selection
Are there any changes?
NO
YES
STOP with Sn and Fi
WTS or WTRS
WTRS
WTS
Apply WTRS Modified Nearest Neighbor Algorithm
Apply WTS Modified Nearest Neighbor Algorithm
Apply 2-Opt Algorithm to improve initial routes

Data Sets & Computational Analysis

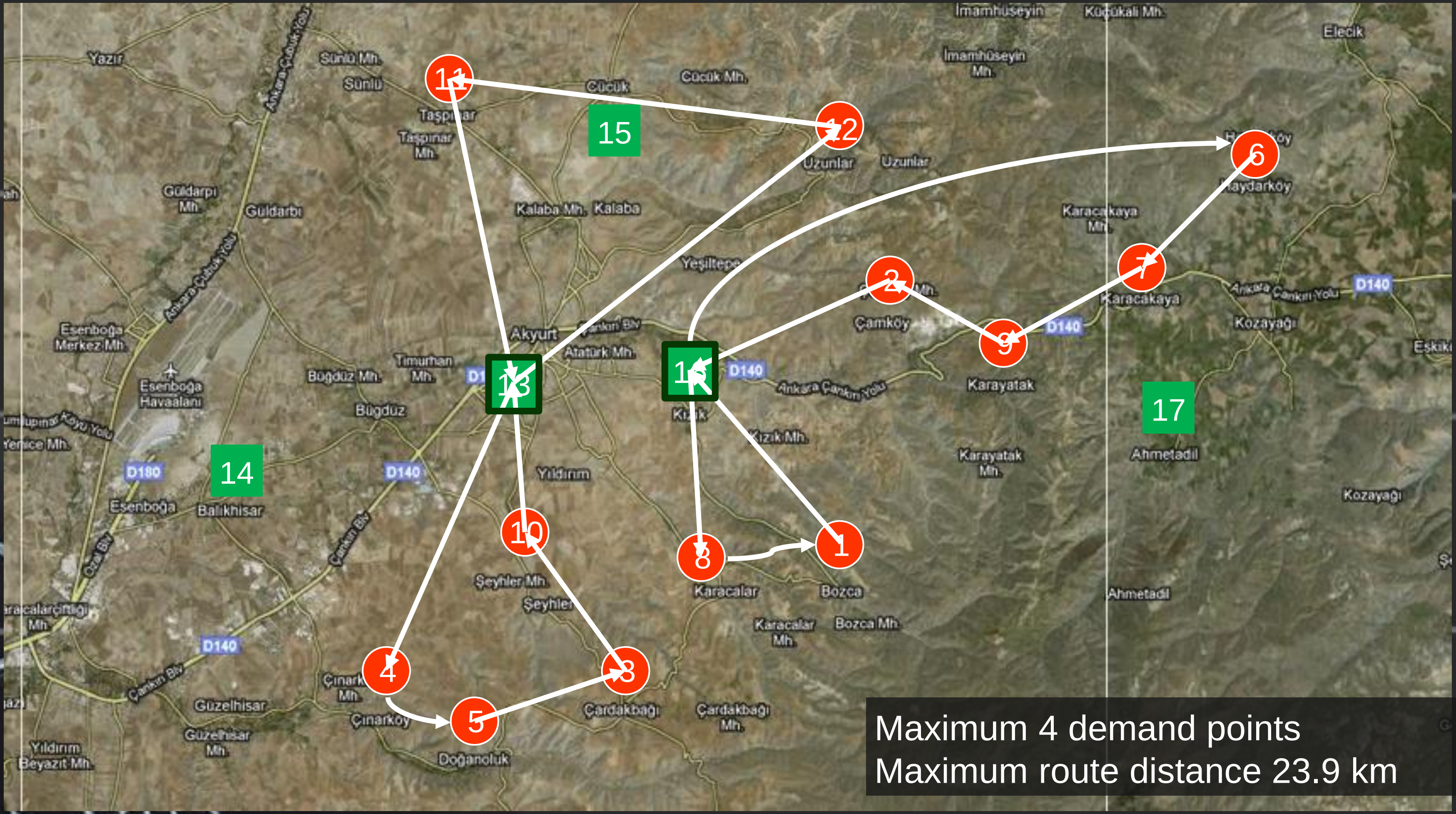


Data | Ankara - Akyurt



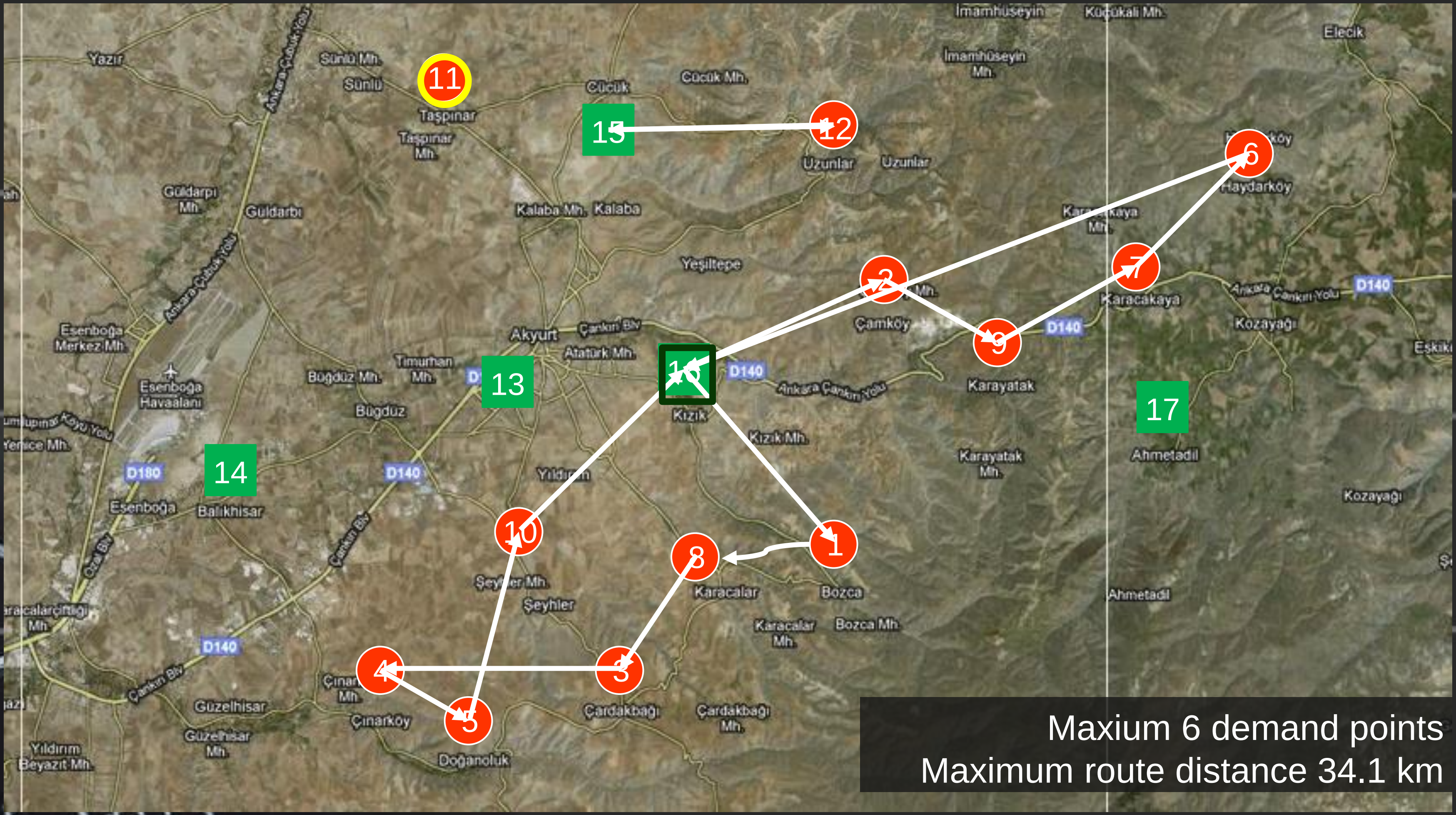
Results

D- WTS
α=4 , γ=2, p=2, B=2



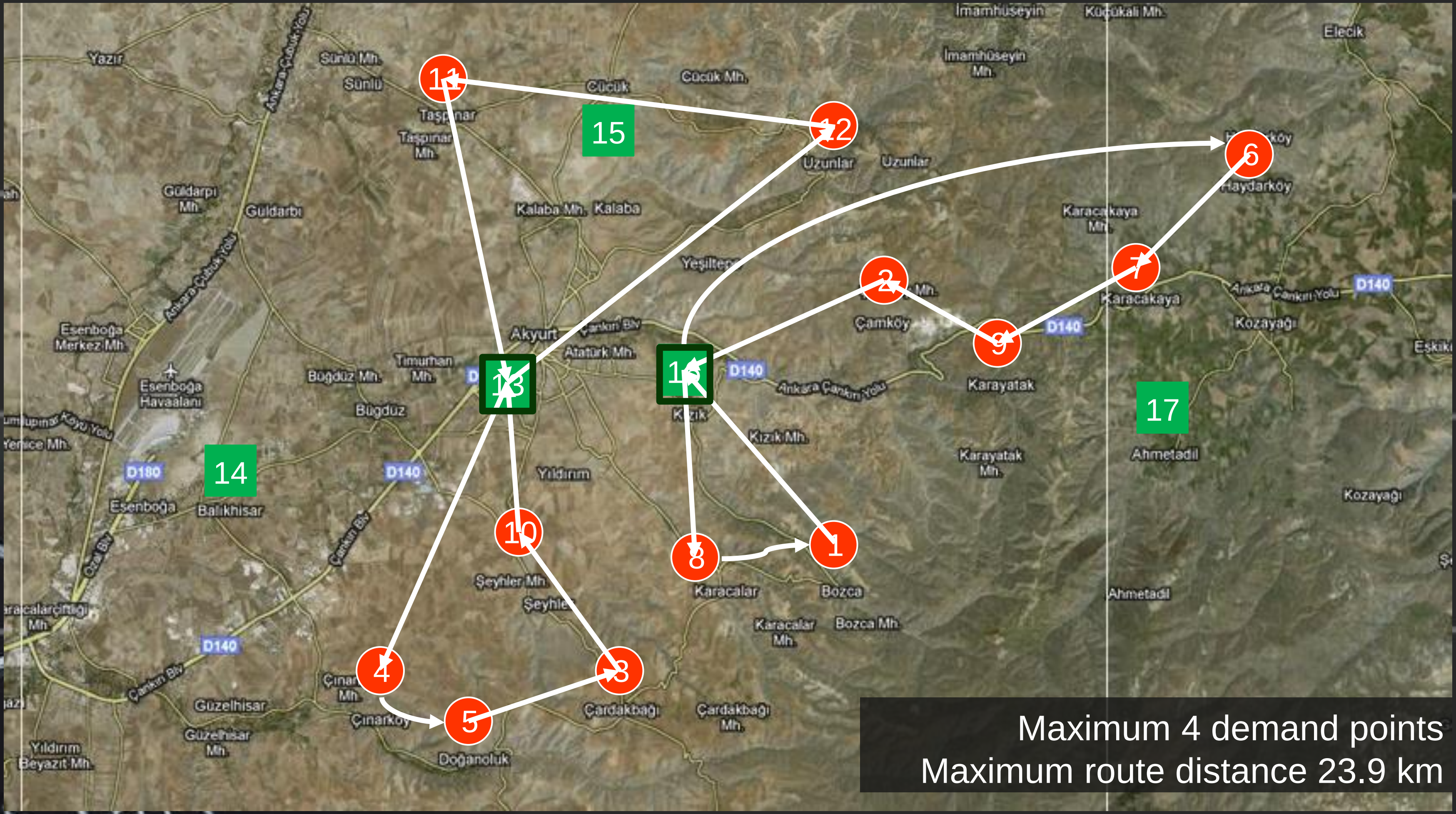
Results

Cum WTRS
α=4 , γ =2, R =50



Results

D-Cum WTRS
α=4 , γ=2, B=2, R =50



Computational Results | Ankara -Akyurt

Instance #	Parameters			Result			
	α	γ	p	z	V	Central School Node Number	S
Instance 16	2	2	1	3312468	1 route	14	-
Instance 17	4	2	1	3260079	1 route	15	11
Instance 18	4	4	1	2836818	1 route	14	1,2,5,10
Instance 19	6	2	1	3260079	1 route	15	11
Instance 20	6	4	1	2836818	1 route	14	1,2,5,10
Instance 21	6	6	1	2607417	1 route	15	1,2,5,6,8,11
Instance 22	2	2	2	3001599	3 routes	14,15	-
Instance 23	4	2	2	2775429	2 routes	14,15	11
Instance 24	4	4	2	2458791	2 routes	14,15	2,5,10,11
Instance 25	6	2	2	2775429	2 routes	14,15	11
Instance 26	6	4	2	2458791	2 routes	14,15	2,5,10,11
Instance 27	6	6	2	2378016	2 routes	15,16	1,2,5,6,8,11
Instance 28	2	2	3	3001599	3 routes	14,15,16	-
Instance 29	4	2	3	2775429	2 routes	14,15,16	11
Instance 30	4	4	3	2458791	2 routes	14,15,17	2,5,10,11
Instance 31	6	2	3	2775429	2 routes	13,14,15	11
Instance 32	6	4	3	2420019	2 routes	14,15,16	1,2,5,8,10,11
Instance 33	6	6	3	2210004	3 routes	14,15,16	1,2,5,6,8,11

Computational Results | Ankara – Akyurt

WTS

P	α	Assignment based Costing Heuristics %	Route based Costing Heuristics %
1	1	10,86965161	2,431906615
1	2	10,86965161	2,431906615
1	4	12,95738539	4,360753221
2	1	22,2681644	10,61104431
2	2	22,2681644	10,61104431
2	4	32,23180993	10,66036998

AVERAGE	18,57747122	6,851170843

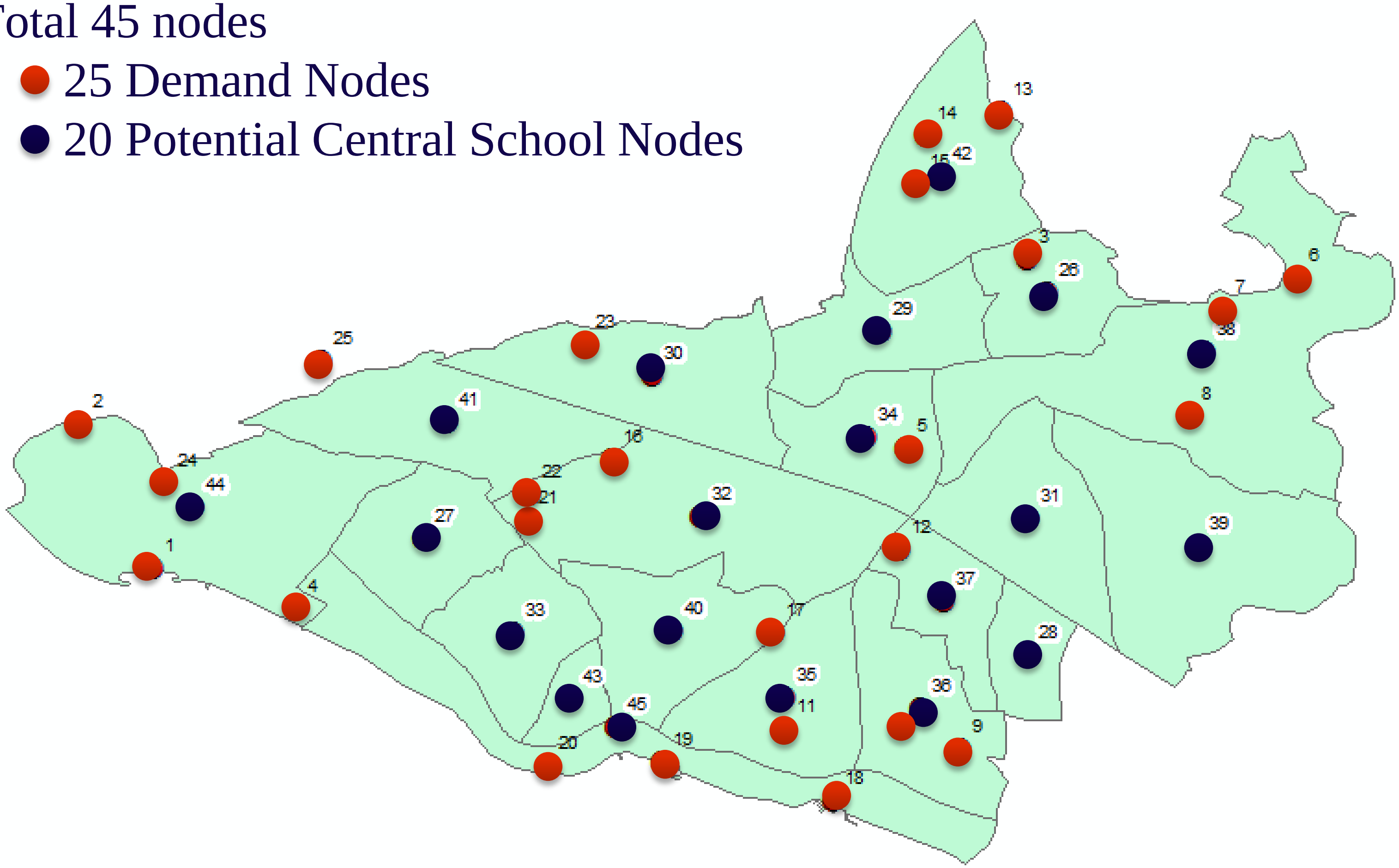
WTRS

P	α	γ	Assignment based Costing Heuristics %	Route based Costing Heuristics %
1	2	2	16,52942761	2,7102149
2	2	2	22,2681644	10,611044
3	2	2	17,9371395	8,9547938
1	4	2	18,40203872	4,3607532
2	4	2	32,23180993	10,66037
3	4	2	27,54784936	8,1490105
1	4	4	31,13283968	9,7949886
2	4	4	48,44693998	12,746468
3	4	4	28,51844667	8,0157687
1	6	2	18,40203872	7,333902
2	6	2	32,23180993	6,6357669
3	6	2	27,54784936	10,82647
1	6	4	31,13283968	8,5420355
2	6	4	48,44693998	13,271929
3	6	4	30,5774872	9,7462458
1	6	6	42,66992967	4,9567445
2	6	6	35,82751336	2,9892145
3	6	6	23,98167605	11,988485

AVERAGE	29,65737443	8,4607892

Data | İstanbul- Kartal

- Total 45 nodes
  - ● 25 Demand Nodes
  - ● 20 Potential Central School Nodes



*

Computational Results | İstanbul – Kartal

			Assignment Based Costing			Route Based Costing		
P	α	γ	Objective	Central School Node Number	S	Objective	Central School Node Number	S
2	2	2	14,1	40,26	1,4,5,7,9,10,12,13,14,15,16,18,20,21,22,24	10,1	42,26	1,4,5,6,7,8,9,10,12,13,14,15,16,18,20,21,22,23,24
3	2	2	13,81	40,26,44	1,2,4,5,7,8,9,10,12,14,15,16,21,22,23,25	8,88	26,42,30	1,2,4,5,6,7,8,8,10,12,13,14,15,16,18,20,21,22,23,25
2	2	-	18,73	40,26	1,4,5,6,8,9,10,14,15,16,18,20,21, 22,24,23	13,97	30,26	1,4,5,6,8,9,10,12,13,14,15,16,18,20,21, 22,24,23
3	2	-	15,17	40,26,44	1,2,4,5,6,12,13,14,15,16,18,20,21,22,23,25	11,21	32,26,42	1,2,4,5,6,7,9,10,12,13,14,15,16,18,20,21,22,23,25

Conclusion

- School Districting Application
- Demand-selective LRP
- Cumulative and Distance-constrained versions
- Two real life scenarios, Akyurt-Ankara and Kartal-İstanbul
- Mathematical Models and Two different heuristics approaches

Thank You for Listening
Questions & Answers