Facility Location

Bahar Y. Kara

Adapted from P. Keskinocak's lecture notes

What is a "Facility"?

Factors influencing location decision

- Locations of customers
- Location of suppliers
- Transportation access
- Real estate costs
- Material costs
- Cost of labor

Expansion capability

Local political conditions

Climate

Weather events

Insurance costs

Locations of competitors

Why do we need optimization models to locate facilities?

Customer locations and demand:

A: (2.0, 2.9),	520 units
B: (3.1, 2.5),	800 units
C: (1.8, 2.2),	540 units
D: (2.4, 1.7),	1,550 units
E: (0.5, 1.6),	790 units
F: (1.7, 0.6),	1,260 units
G: (3.3, 1.4),	2,050 units

Locate a distribution center to minimize the weighted distance from customers (center-of mass)

Measuring Distances

- "Manhattan distance" (1-norm)
  - Best for cities with perpendicular streets
  - N-S distance + E-W distance

$$L_1 = |x_1 - x_2| + |y_1 - y_2|$$

- "As the crow flies" (2-norm)
  - Best for long distances and highways

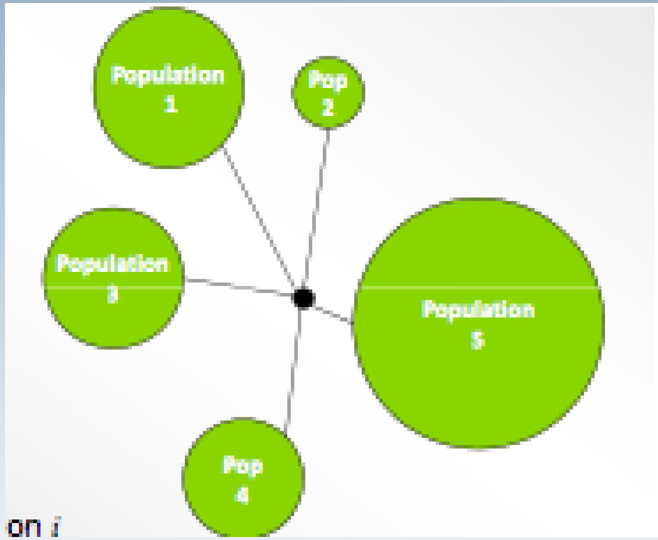
$$L_2 = \sqrt[2]{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

- Larger of N-S and E-W distances (∞-norm)
  - Useful for automatic warehouses

$$L_\infty = \max(|x_1 - x_2|, |y_1 - y_2|)$$

Center-of-mass

- Center-of-mass location = The weighted average location of a set of populations
- Weights can be:
  - Population size
  - Demand
  - Importance
  - Severity of need



$$(\bar{x},\bar{y}) = \frac{\sum_i d_i(x_i, y_i)}{\sum_i d_i}$$

d_i = weight of population *i*
x_i = x-coordinate of population *i*
y_i = y-coordinate of population *i*

Example

- Center of Mass:

$$\bar{x} = \frac{x_A d_A + x_B d_B + x_C d_C + x_D d_D + x_E d_E + x_F d_F + x_G d_G}{d_A + d_B + d_C + d_D + d_E + d_F}$$

$$\bar{x} = \frac{x_A d_A + x_B d_B + x_C d_C + x_D d_D + x_E d_E + x_F d_F + x_G d_G}{d_A + d_B + d_C + d_D + d_E + d_F + d_G}$$

$$= \frac{(2.0*520)+(3.1*800)+(1.8*540)+(2.4*1550)+(0.5*790)+(1.7*1260)+(3.3*2050)}{520+800+540+1550+790+1260+2050}$$

$$= 2.3$$

$$\bar{y} = \frac{(2.9*520)+(2.5*800)+(2.2*540)+(1.7*1550)+(1.6*790)+(0.6*1260)+(1.4*2050)}{520+800+540+1550+790+1260+2050}$$

$$= 1.6$$

Example (cont.)

- Least-distance facility located at (2.3, 1.6)
- What could be wrong with this location?
  - Cost of land
  - Availability of land
  - Traffic congestion to/from facility
  - No information about tax structure
  - Zoning

Example (cont.)

Candidate locations, costs:

1: (2.3, 1.6), $30M + $50/unit

2: (1.3, 2.5), $15M + $40/unit

3: (1.9, 3.7), $12M + $30/unit

4: (3.7, 3.2), $15M + $35/unit

5: (0.8, 0.3), $10M + $40/unit

Assumptions:

- Travel costs are $1/mile-unit
- Demand fulfilled weekly
- Each unit square is 8x8 miles

Developing the optimization model

- Decisions to make:
  - Whether to open each candidate location?
  - How much to ship to each customer from each opened facility

- Optimization model:
  - *Minimize cost*
    - All demand must be fulfilled
    - Limit number of facilities to be opened
    - Only ship from open facilities

Notes

Facility Location Integer Programming Sample Formulation

- Objective Function
  - Can be minimize cost, delivery time, maximize impact etc.
- Decision Variables:
  - Binary variables indicating whether each facility i should be opened
  - Amount of each item to be held at each facility
  - Amount of each item to be received from each supplier to each facility
  - Amount of each item to be sent from each facility to each customer

Facility Location Integer Programming Sample Formulation (2)

- Inputs: –
  - Set of possible locations to open facilities
  - Set of suppliers
  - Set of customers
  - Cost or time along each transport route
  - Set of products to be supplied to customers
  - Demand of each customer for each product
  - Max number of facilities to open
  - Max inventory space available at each facility

Facility Location Integer Programming Sample Formulation (3)

- Minimize or maximize
- Subject to
  - Customer demand is satisfied
  - Flow out of each facility = Flow into each facility
  - Max number of facilities (budget)
  - Max inventory space is not exceeded
  - Inventory only held at open facilities

Many different objective functions

- Maximize profit (common in for-profit problems)
- Minimize cost under a minimum acceptable service level (eg minimum number of facilities, a certain percentage of demand met)
- Maximize service level under some constraint (ie budget)
- Minimize average response time

Many different objectives functions

- Common healthcare objectives
  - Minimize cost (common in for-profit healthcare)
  - Maximize benefit to health (can be minimize negative health outcomes and maximize positive health outcomes)
  - Maximize equitability or fairness
- Multiple objectives

Facility Location Literature

Facility Location Literature

- P-median
- Location with fixed costs
- Covering problems
- Center problems

Notes