



The Demand- Selective Location Routing Problem: the School Districting Application



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School Districting

Eight years primary education is obligatory

In some districts, there aren't any primary school

School Districting Program

Students in these districts are transported to districts with primary schools

Central Schools



School Districting

1989-90 → 5 Central Schools and 305 students.

2009-10 → 80 cities with 667,475 students

Now → 5,754 central schools serving students from 39,559 districts in Turkey

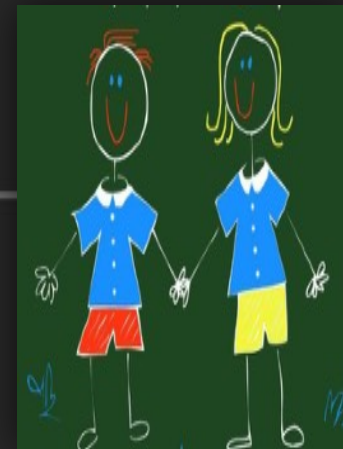
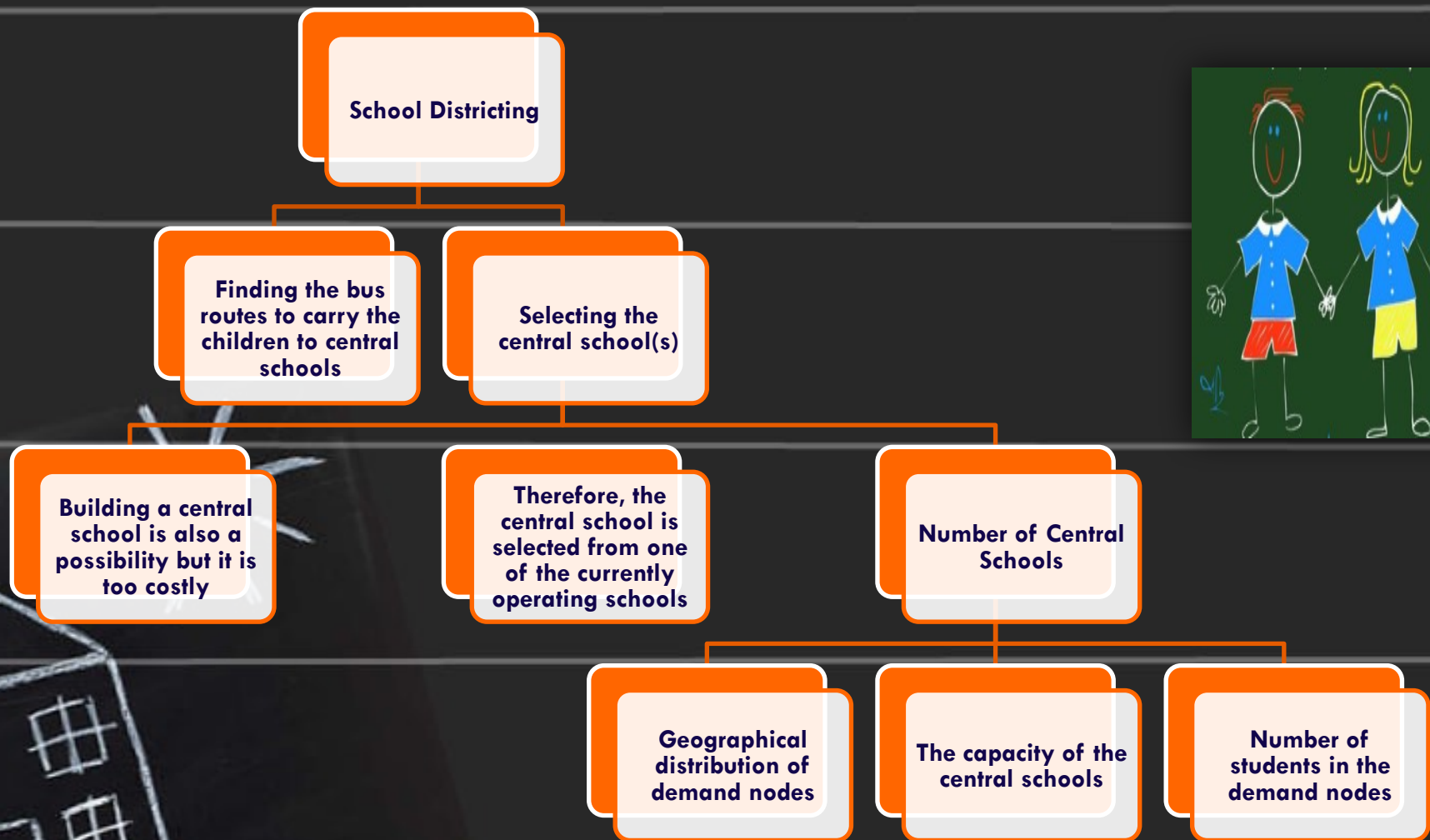
Mathematical models are developed

Heuristic approaches for minimizing the cost of school districting application

Real Life Scenarios

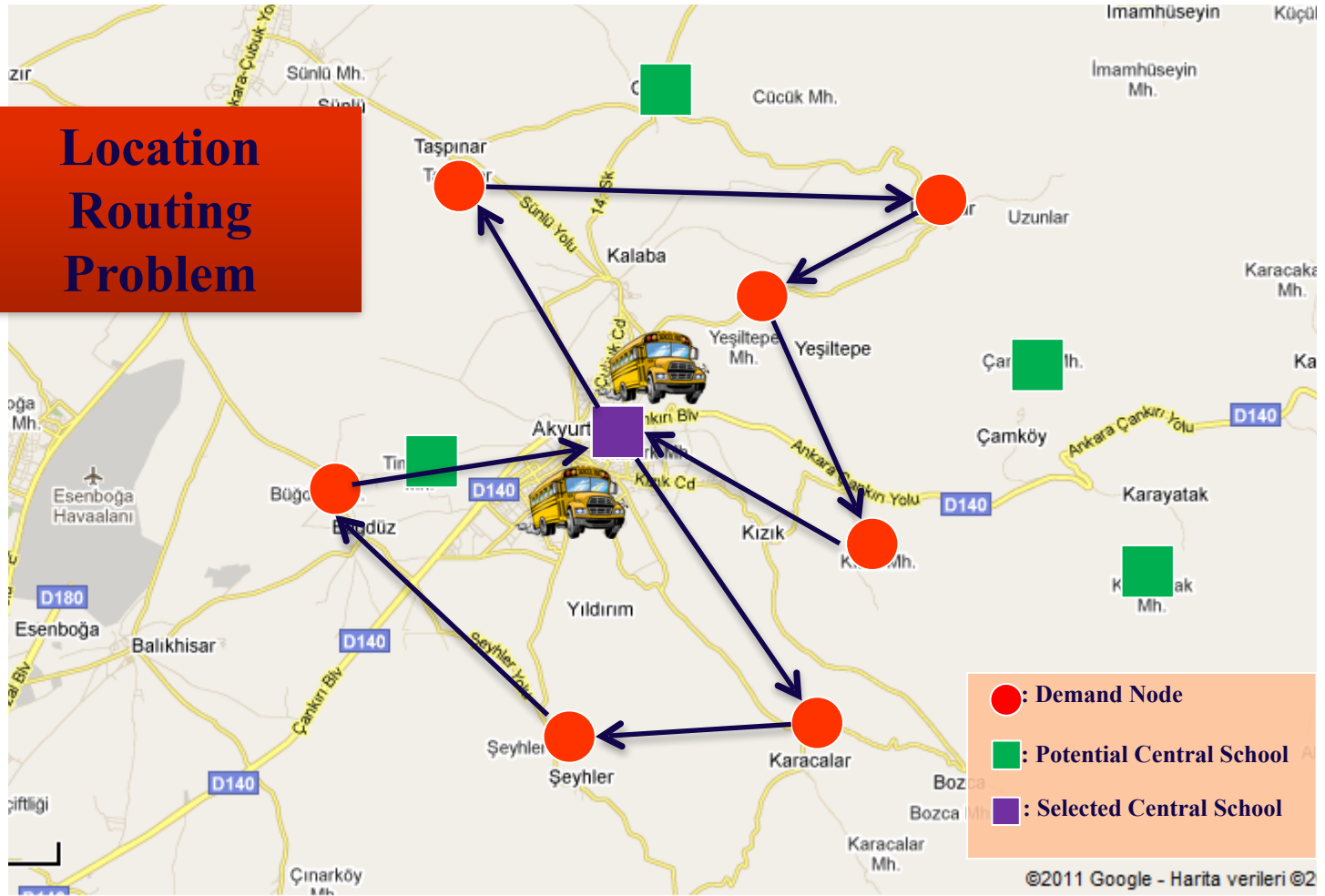


School Districting



School Districting

**Location
Routing
Problem**



School Districting

Location Routing Problem

the selection of central
school location

assignment of demand
sites to central school

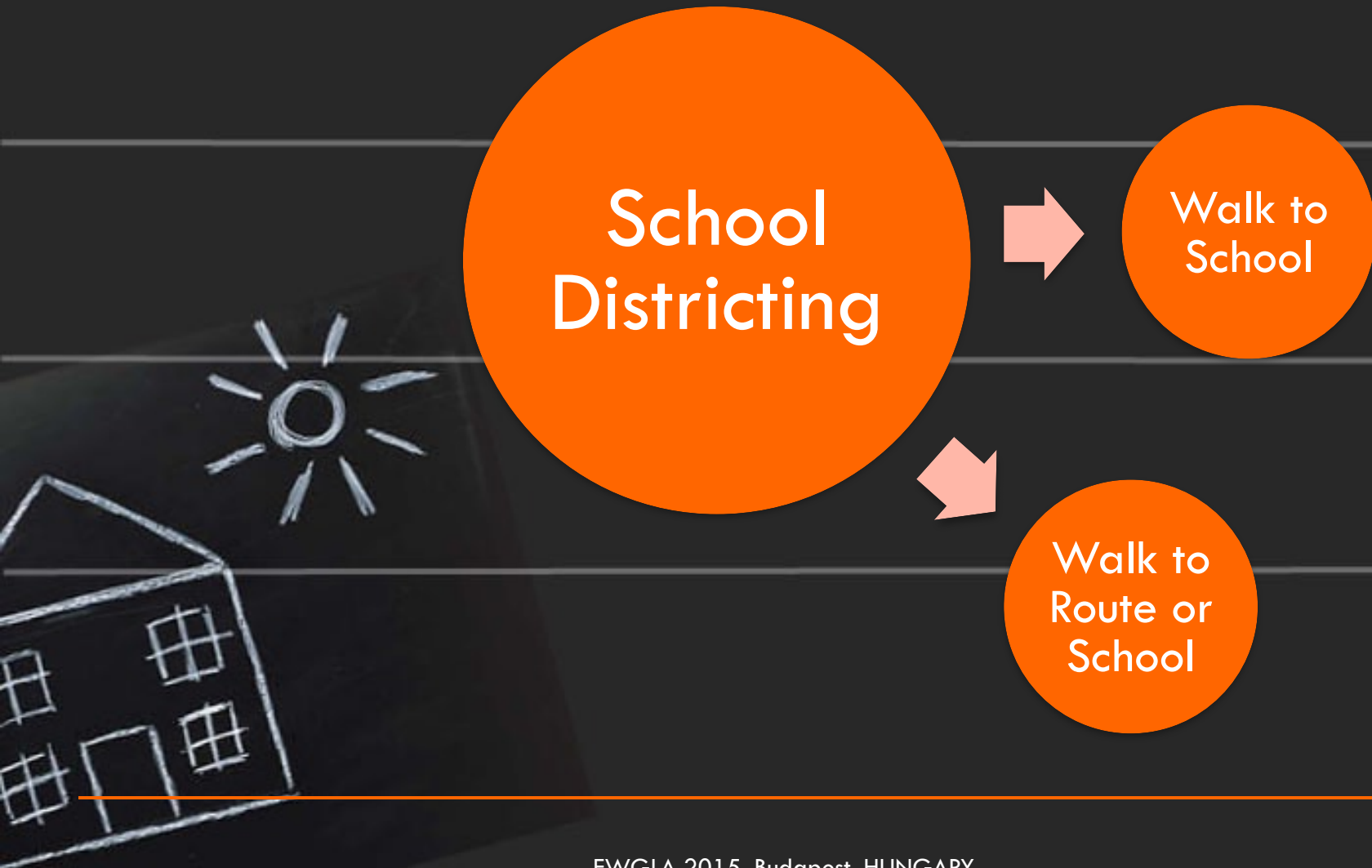
transportation routes
between demand nodes



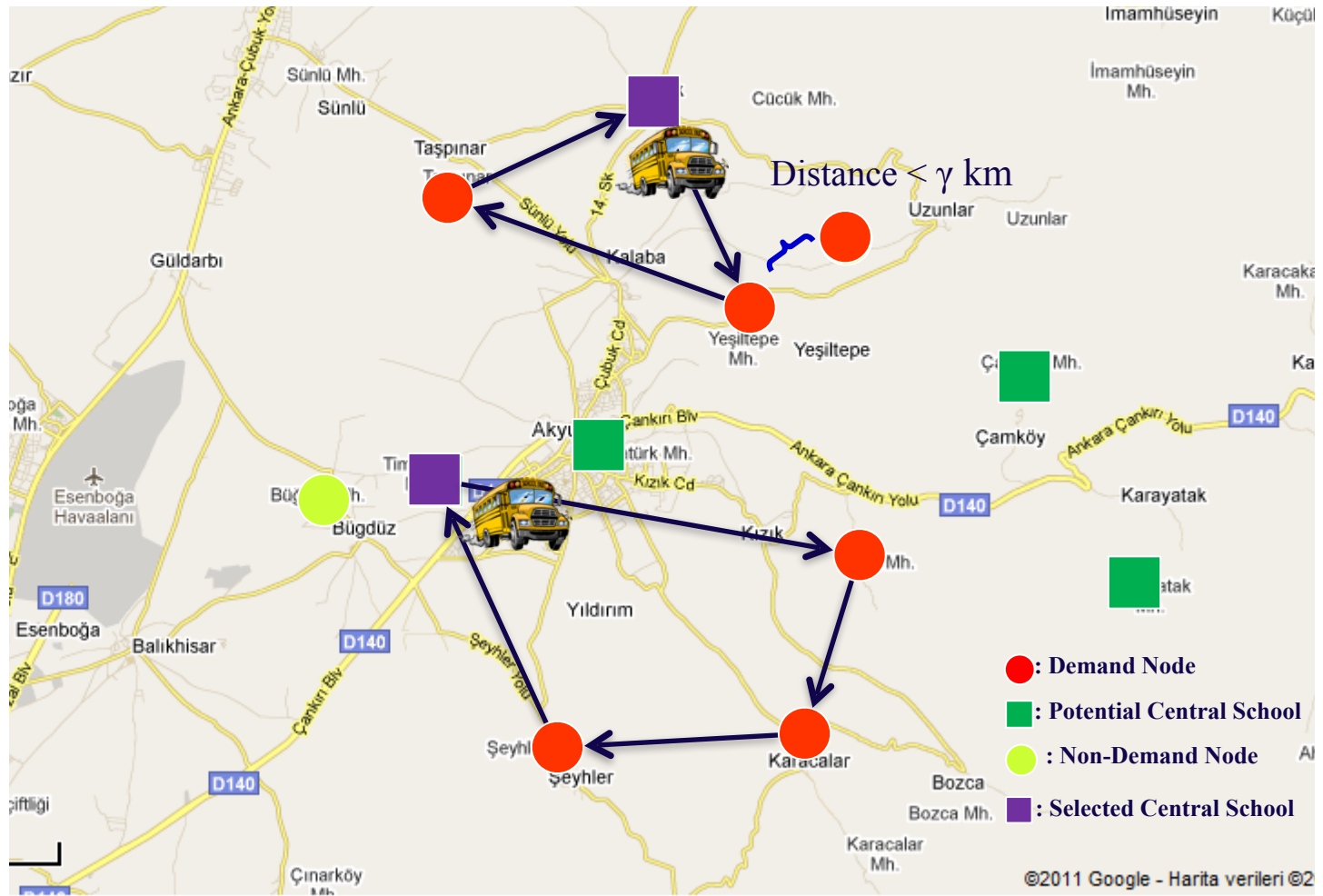
Demand Selective Location Routing Problem

the students at a demand
site may walk directly to
central school or to other
demand sites to be picked
up.

School Districting



School Districting I Walk to Route or School



Literature Survey

Location Routing Problem

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Heuristics

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Mathematical Model | Walk To School

$$s_i = \begin{cases} 1, & \text{if demand node } i \in D \text{ needs to be visited by a vehicle} \\ 0, & \text{otherwise} \end{cases}$$

$$x_j = \begin{cases} 1, & \text{if candidate location } j \in C \text{ is selected as a central school location} \\ 0, & \text{otherwise} \end{cases}$$

$$z_{ijk} = \begin{cases} 1, & \text{if node } i \text{ immediately precedes node } j \text{ using bus of } k^{\text{th}} \text{ central school; } i, j \in N, k \in C \\ 0, & \text{otherwise} \end{cases}$$

y_{ijk} : Number of students on a bus of k^{th} central school, exiting from node i right before node j ;
 $i \in D; j, k \in C$

SD_j : Set of all demand nodes that the distance to $j \in D$ is less than or equal to α km

$$SD_j = \{ i: I_{ij} \leq \alpha, i \in D \}$$

SC_i : Set of all candidate central school nodes that the distance to node $i \in N$ less than or equal to α km, $SC_i = \{ j: I_{ij} \leq \alpha, j \in C \}$.

$$SR_i = \{ h: h \in D, l_{ih} \leq \gamma \}.$$

Model

$$\text{Min} \sum_{i \in N} \sum_{j \in N} \sum_{k \in C} c_{ij} z_{ijk} \quad (1)$$

s.t.

$$\sum_{k \in C} \sum_{j \in N} z_{ijk} = s_i \quad \forall i \in D \quad (2)$$

$$1 - x_j \geq s_i \quad \forall i \in SD_j, \forall j \in C \quad (3)$$

$$1 - \sum_{j \in SC_i} (x_j) \leq s_i \quad \forall i \in D \quad (4)$$

$$\sum_{j \in C} x_j = p \quad (5)$$

Model

$$\sum_{j \in N} \sum_{k \in C} z_{jik} = \sum_{j \in N} \sum_{k \in C} z_{ijk} \quad \forall j \in D \quad (6)$$

$$\sum_{i \in D} z_{ijj} = \sum_{i \in D} z_{jij} \quad \forall j \in C \quad (7)$$

$$z_{ijk} \leq x_k \quad \forall i, j \in N, \forall k \in C \quad (8)$$

$$y_{ijk} \geq z_{ijk} \quad \forall i, j \in N, \forall k \in C \quad (9)$$

$$y_{ijj} = z_{jij} \quad \forall i \in D, \forall j \in C \quad (10)$$

$$\sum_{j \in N} y_{ijk} = \sum_{r \in N} y_{rik} + \sum_{j \in N} q_i z_{ijk} \quad \forall i \in D, \forall k \in C \quad (11)$$

$$y_{ijk} \leq QV z_{ijk} \quad \forall i, j \in N, \forall k \in C \quad (12)$$

$$\sum_{i \in D} y_{ikk} \leq QS \quad \forall k \in C \quad (13)$$

$$x_j, s_i, z_{ijk} \in 0, 1; y_{ijk} \geq 0, \text{ integer} \quad \forall i \in D, \forall j, k \in C \quad (14)$$

EXTENSIONS



Model I Walk to Route or School

Min (1)

s.t.

(2), (3), (5) - (14)

$$1 - \sum_{j \in \text{Candidate}_i} (x_j) \leq s_i$$

$\forall i \in I$ 

$$1 - \sum_{j \in SC_h} x_j - \sum_{m \in N} \sum_{k \in J} \sum_{i \in SR_h} z_{mik} \leq s_h$$

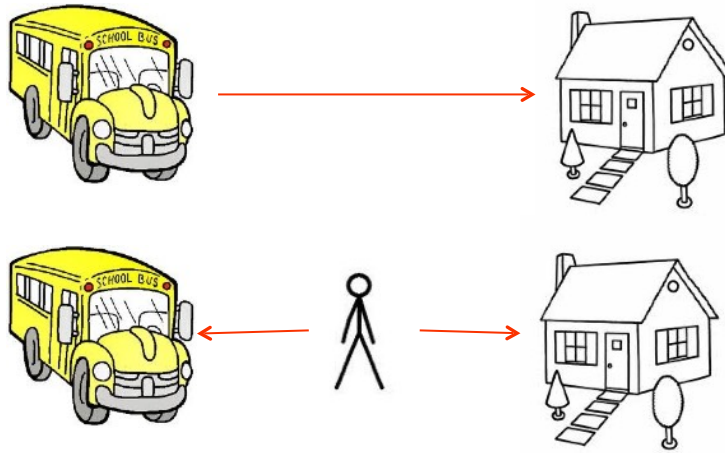
$\forall h \in D$ (4')

District must be visited if and only if there is no central school within α km, and no demand nodes which is visited by a node within γ km.

Model Extension I Distance Constrained Walk to Route or School

- Students

- Central School
- + Another District
- + **Travel Distance Constraint**



Model Extension I Distance Constrained Walk to Route or School

Min (1)

s.t.

(2), (3), (4'), (5) - (14)

$$t_{ijk} \leq B z_{ijk} \quad \forall i, j \in N, \forall k \in C \quad (15)$$

$$\sum_{j \in N} t_{ijk} = \sum_{r \in N} t_{rik} + \sum_{j \in N} l_{ij} z_{ijk} \quad \forall i \in D, \forall k \in C \quad (16)$$

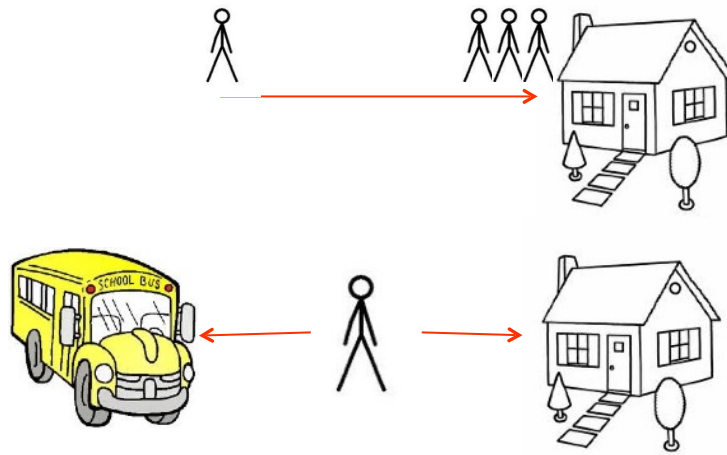
$$t_{ijk} \geq 0 \quad \forall i, j \in N, \forall k \in C \quad (17)$$



Model Extension I Cumulative Walk to Route or School

- Students

- Central School
- + Another District
- + Travel Distance Constraint
- **The objective is not related to buses, but to the number of students**



Model Extension I Cumulative Walk to Route or School



Model Extension I Cumulative Walk to Route or School

$$\text{Min } \sum_{i \in N} \sum_{j \in N} \sum_{k \in C} c'_{ij} y_{ijk} \quad (1)$$

s.t.

(2), (3), (4'), (5) – (14)

$$\sum_{i \in D} z_{jij} \leq R x_j \quad \forall j \in C \quad (18)$$

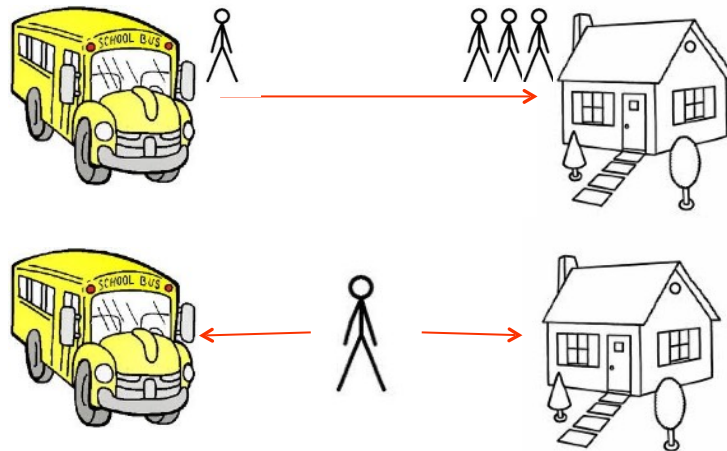
$$\sum_{i \in D} z_{ijj} \leq R x_j \quad \forall j \in C \quad (19)$$



Model Extension I D-Cum WTRS

- Students

- Central School
- + Another District
- + Travel Distance Constraint
- The objective is not related to buses, but to the number of students



Model Extension I D-Cum WTRS

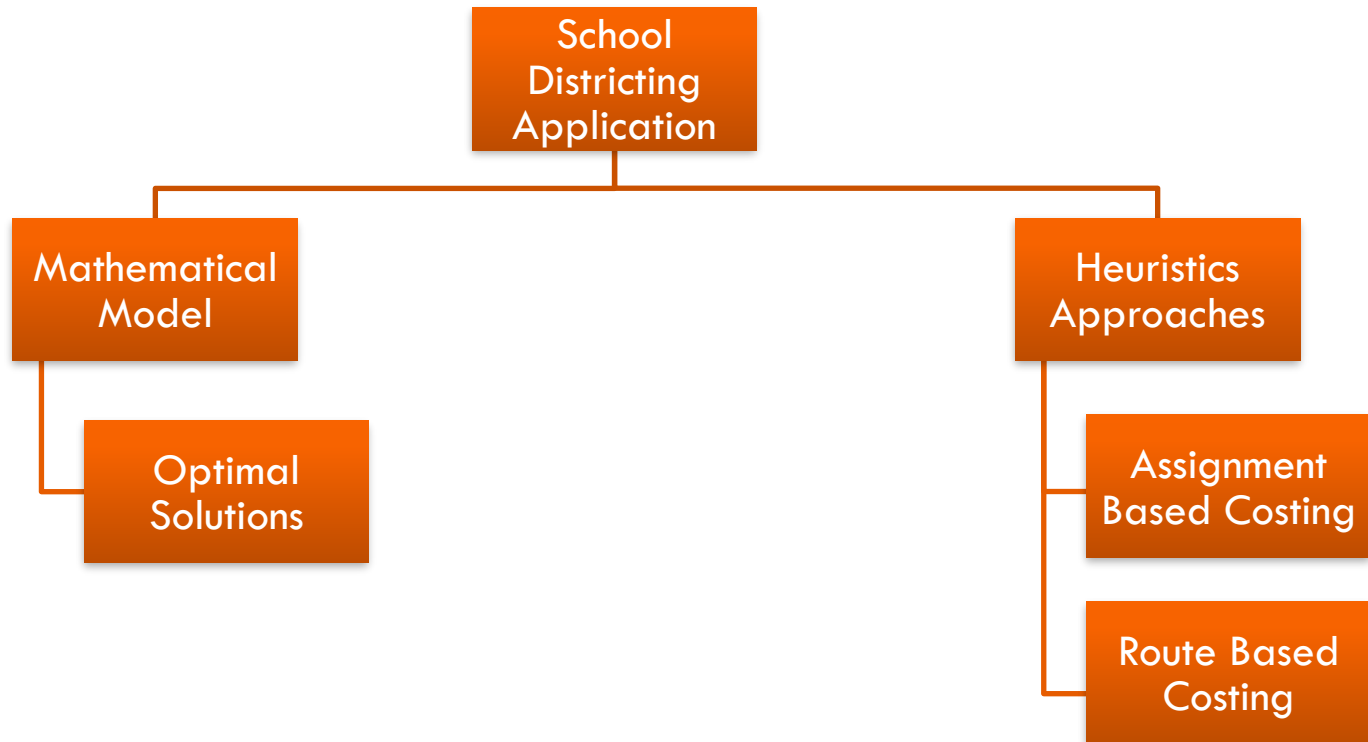
Min (1')

s.t.

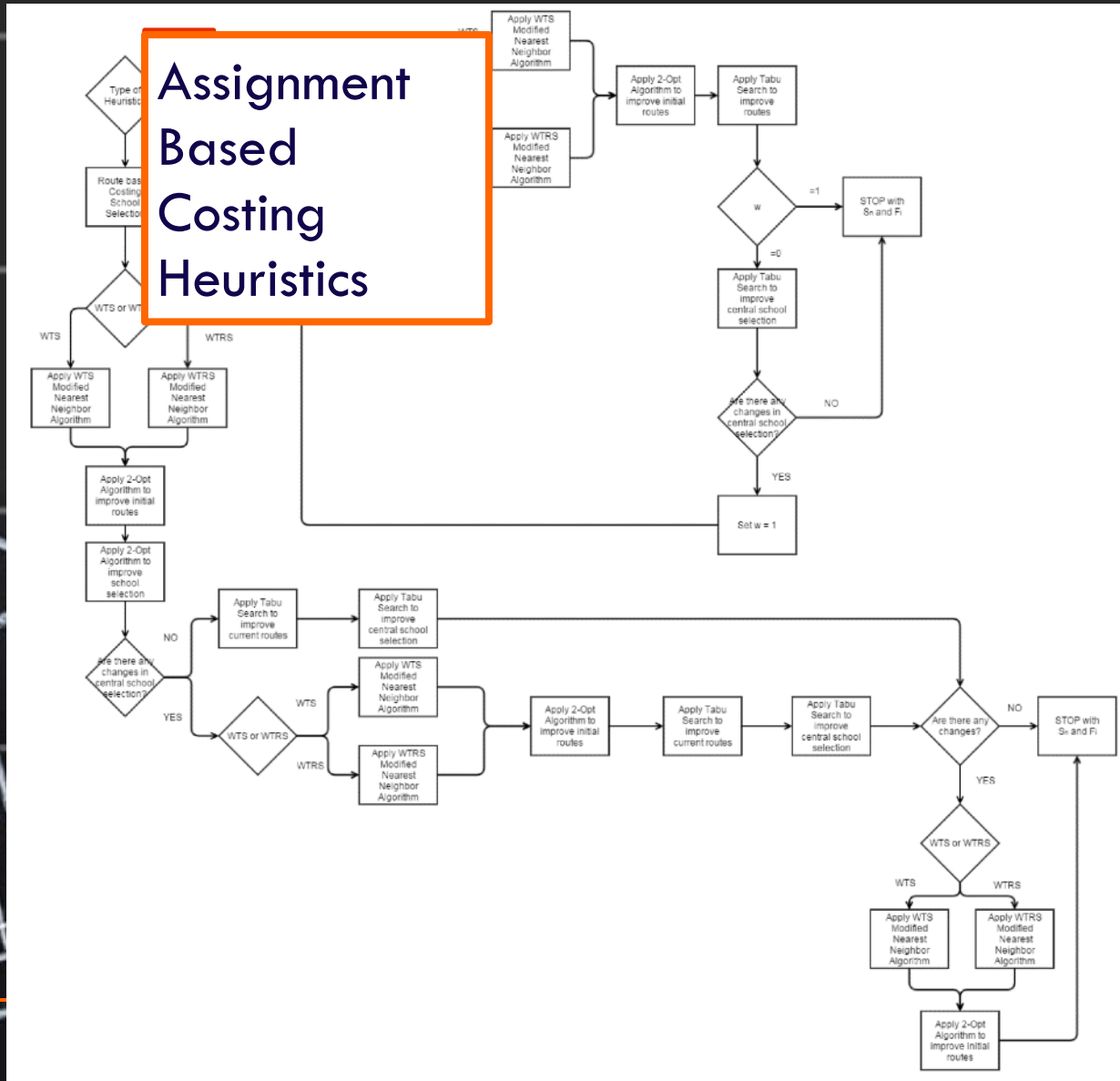
(2), (3), (4'), (5) – (19)



Solution Methodologies

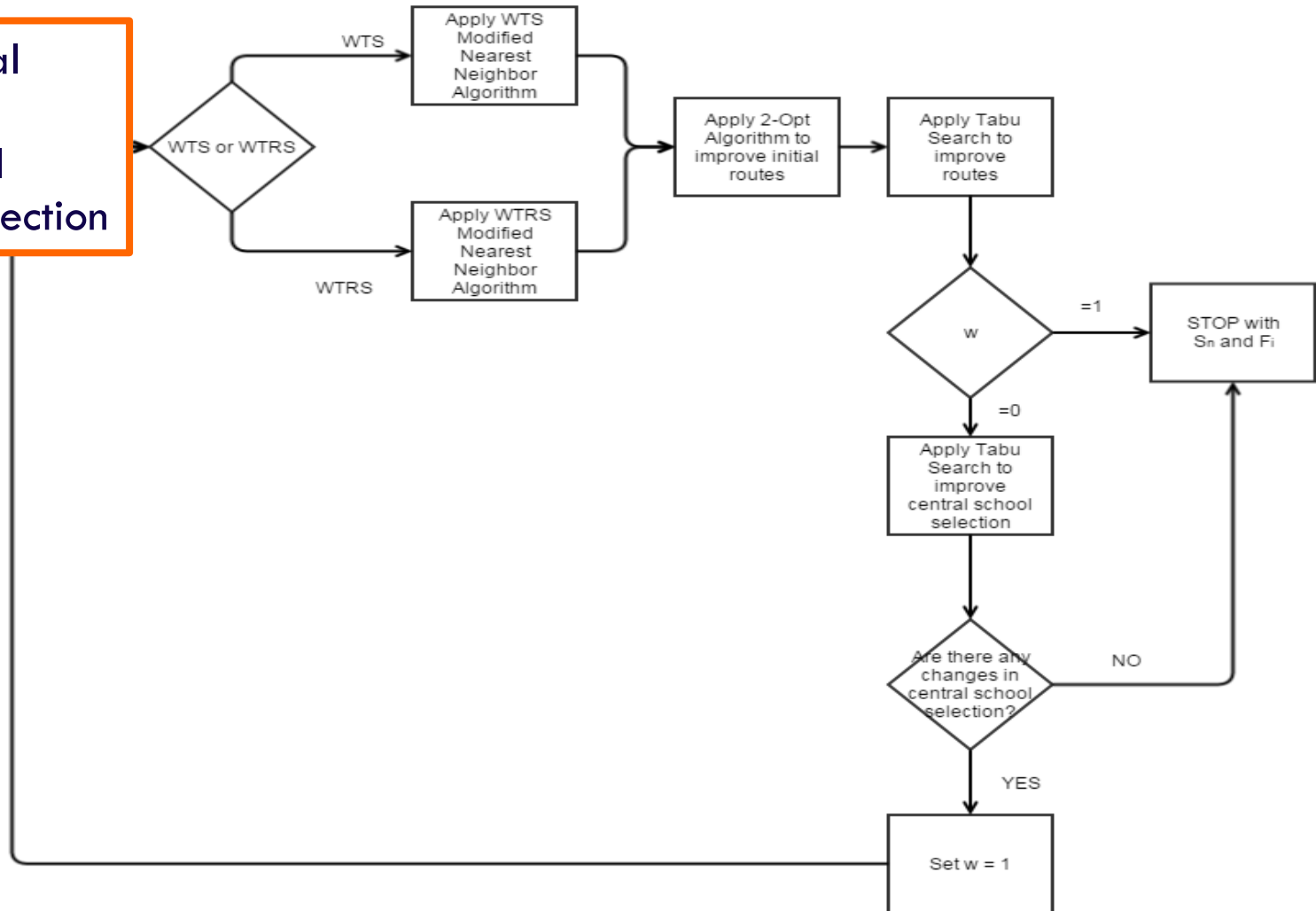


Heuristics



Heuristics

Incremental Greedy Sequential School Selection



Heuristics | Incremental Greedy Sequential School Selection

D: Set of districts

PS: Set of possible central schools

PSC: Copy set of possible central schools

S^n : Set of Selected Central Schools

$CS^P = \{ \text{all } P\text{-tuple of } i \in \text{PSC} \}$

Q^P : Set of cost for CS^P

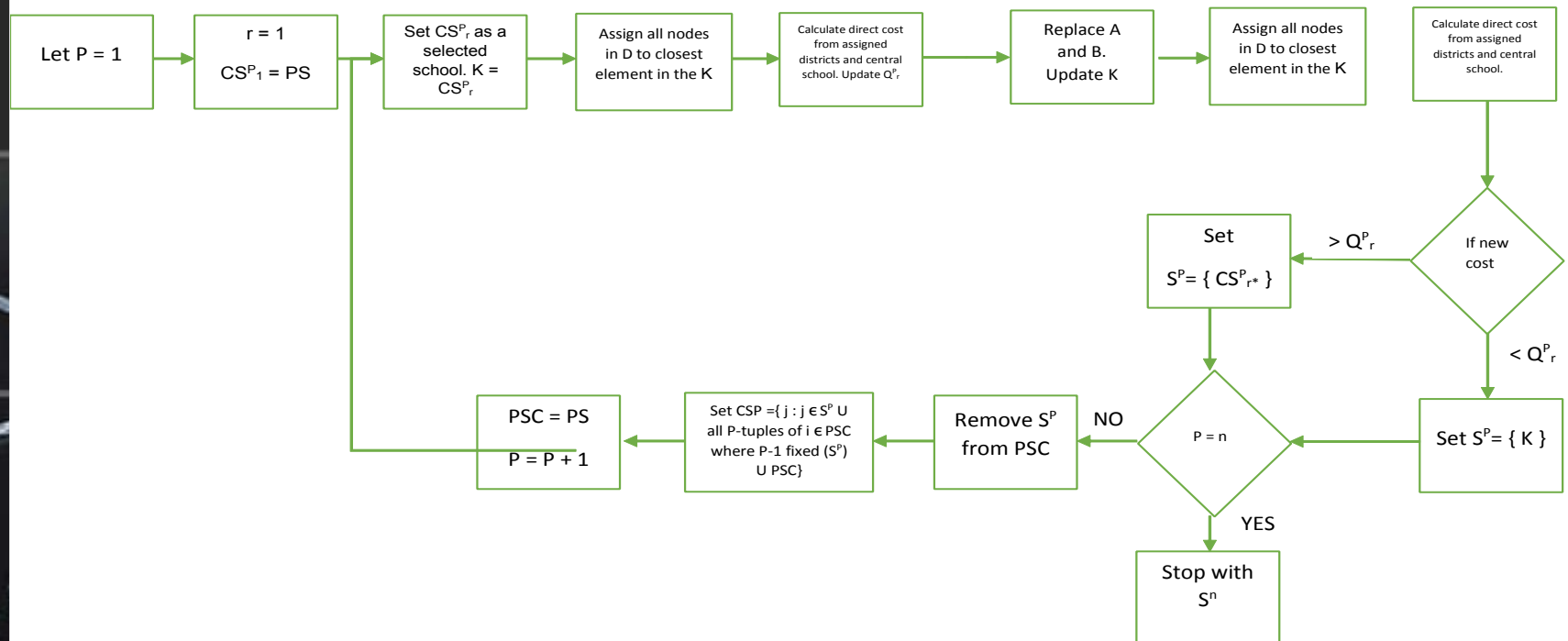
CS_r^P represents the r^{th} P-tuple of CS^P

Q_r^P represents the r^{th} element of Q^P

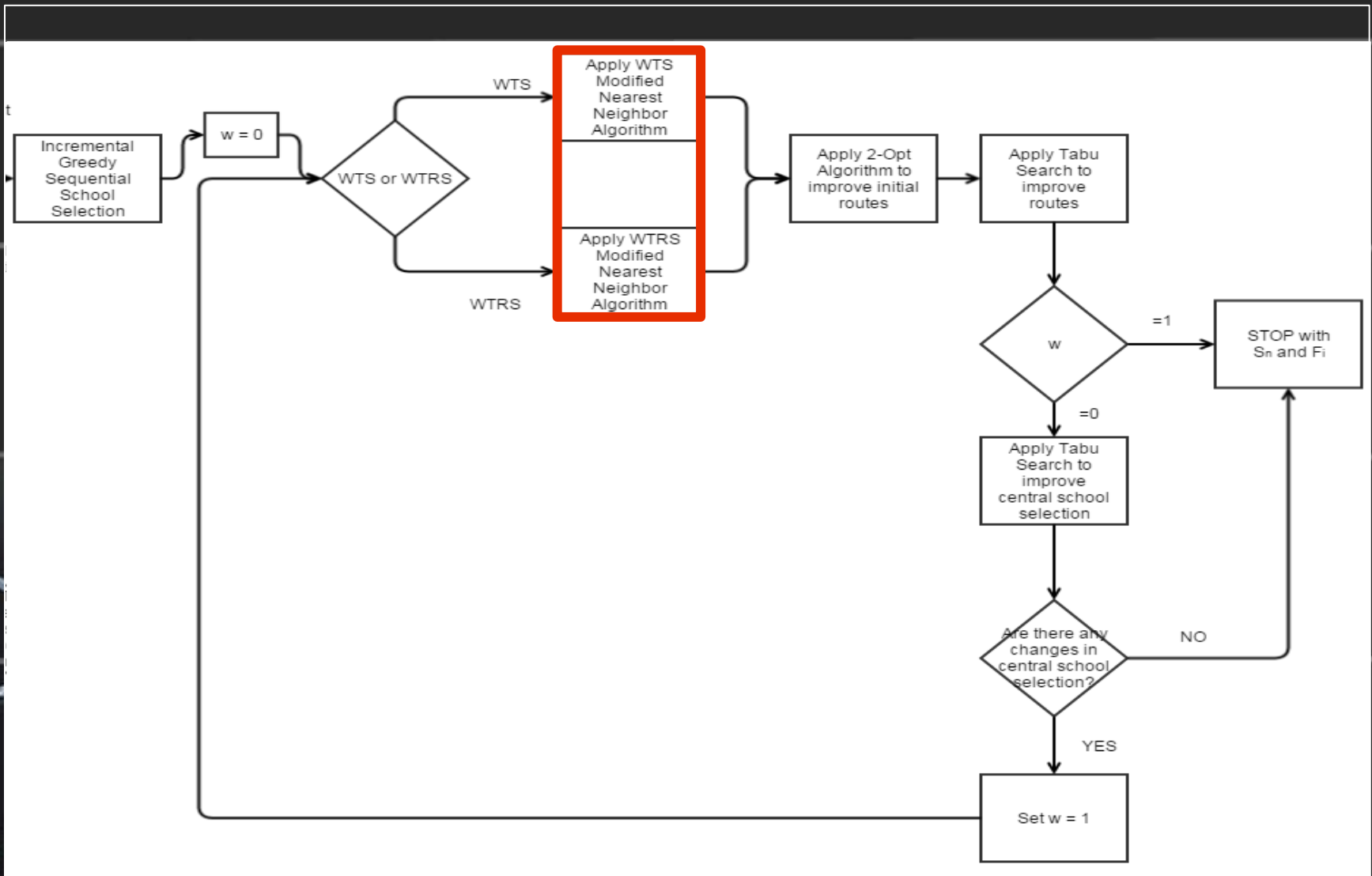
$|CS^P| = R$

A: Set of central schools in CS_r^P

B: Set of central schools not in CS_r^P



Heuristics



Heuristics | Nearest Neighbour Algorithm

W_i : Set of WTS

WR_i : Set of WTRS

R_i : Set of districts which are assigned to be picked up by school bus

F_i : Set of final routes

l_{ij} : Set of distances between i th school and j th district

K_{mj} : Set of distances between i th district and j th district

w : tabu search decision variable

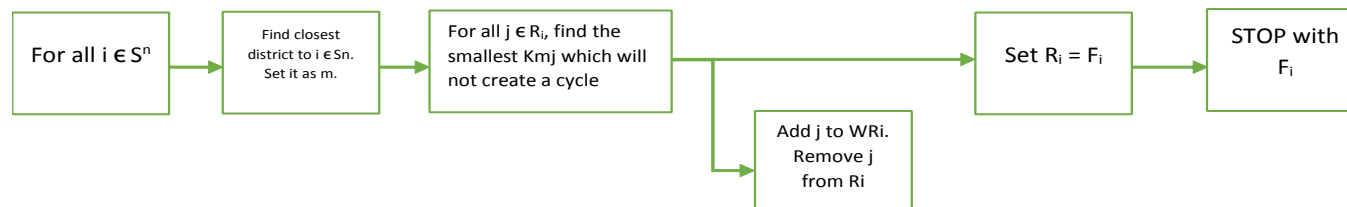
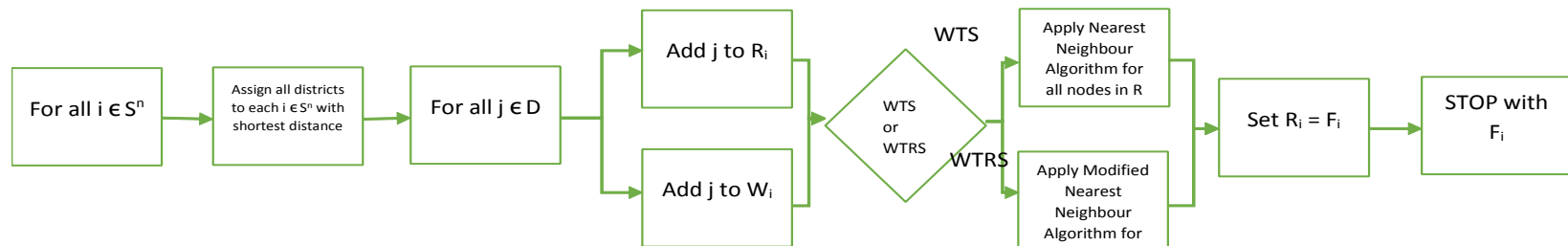
$W_1 = \{\}$

$WR_1 = \{\}$

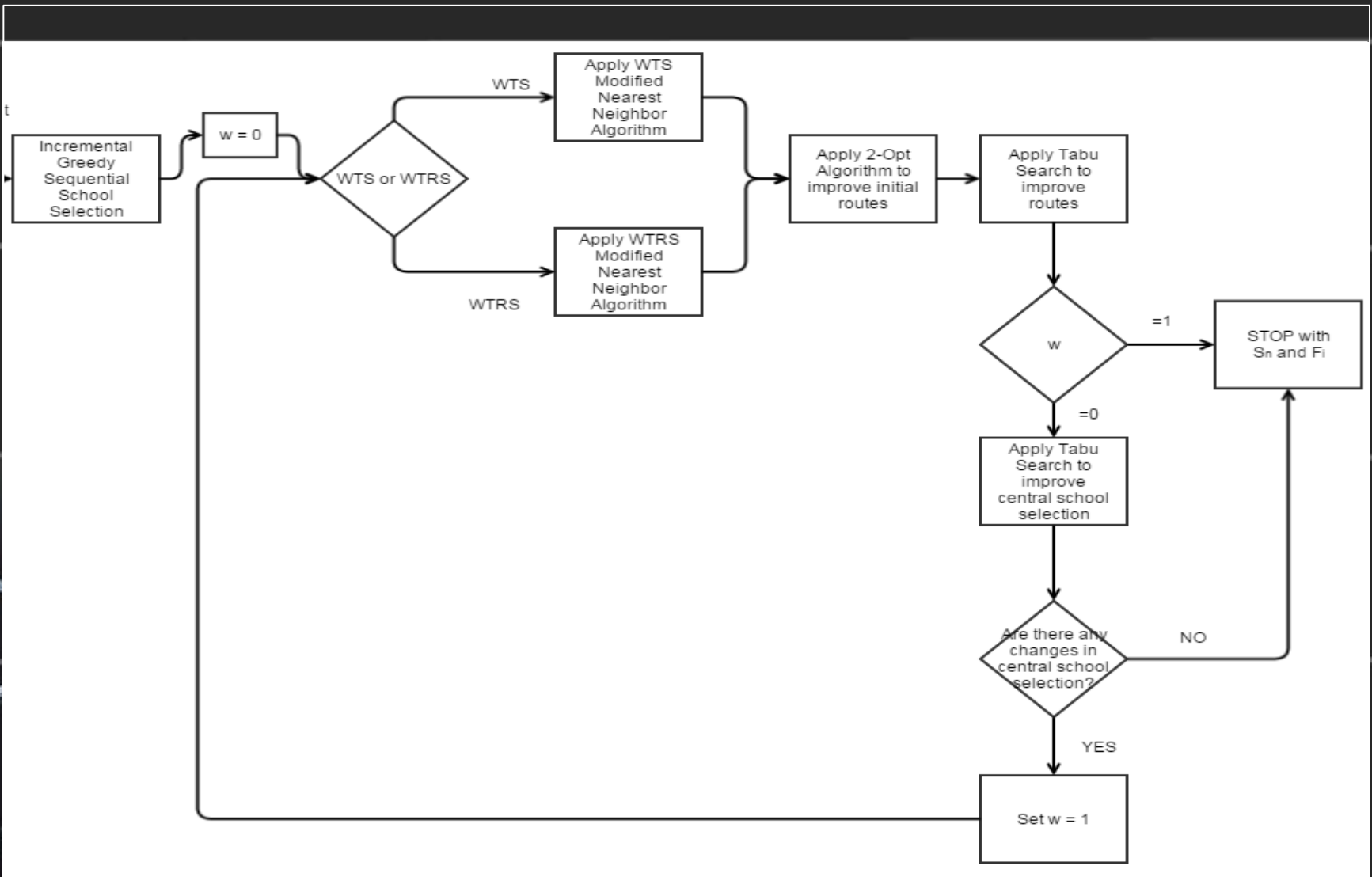
$R_1 = \{\}$

$F_1 = \{\}$

$w = 0$

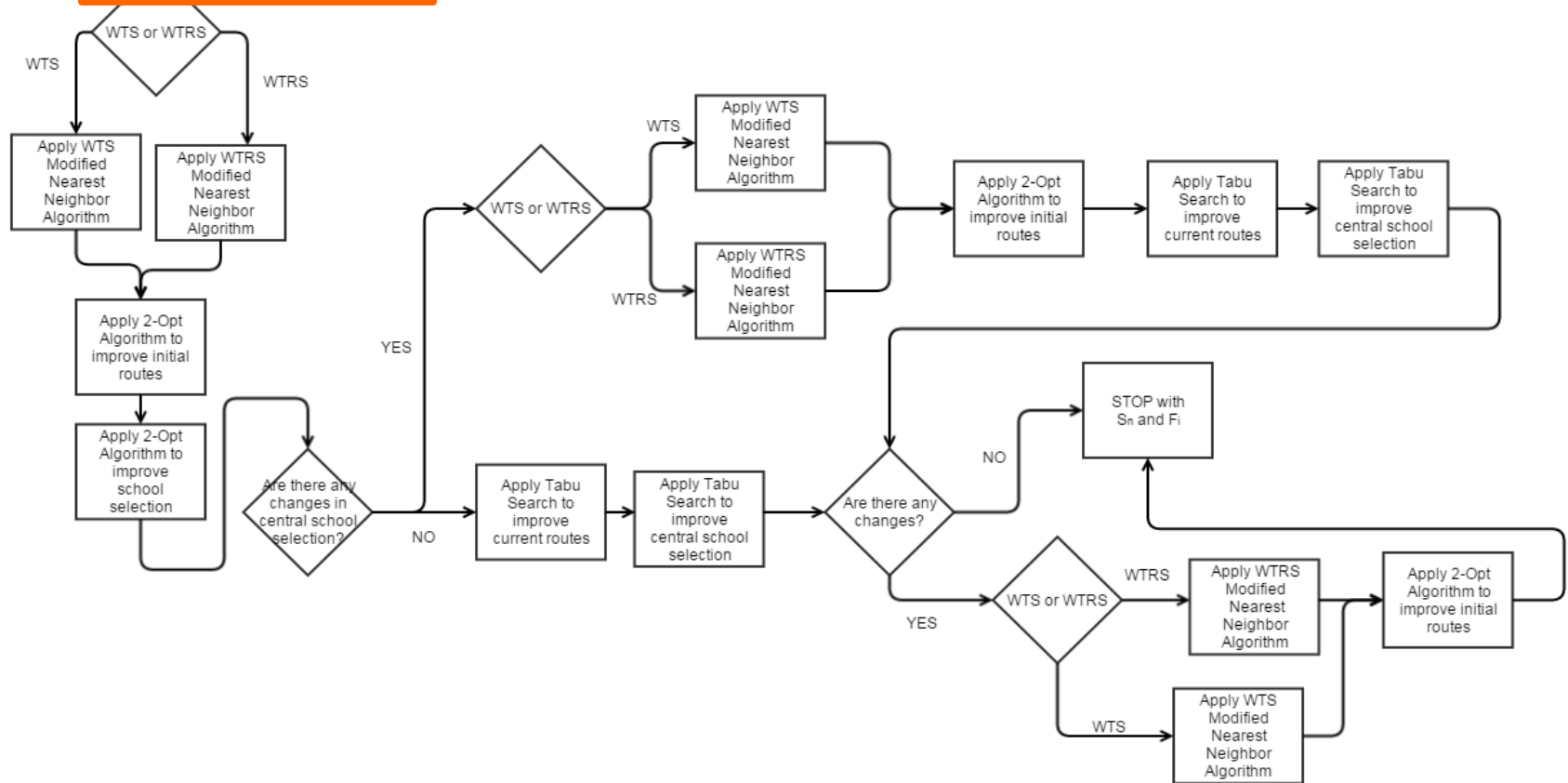


Heuristics



Heuristics

Route Based Costing School Selection



Heuristics | Route Based Costing School Selection

D: Set of districts

PS: Set of possible central schools

PSC: Copy set of possible central schools

S^n : Set of Selected Central Schools

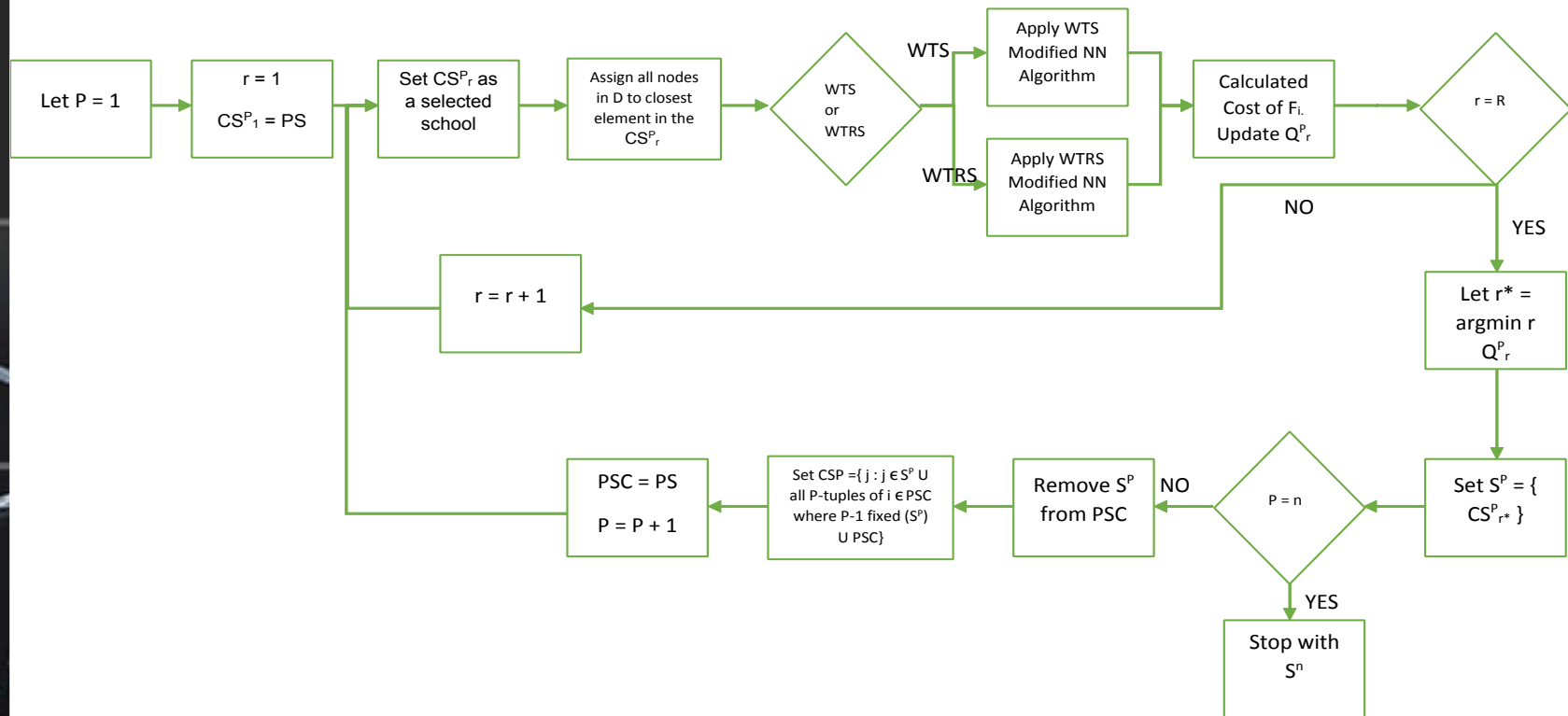
$CS^P = \{\text{all } P\text{-tuple of } i \in \text{PSC}\}$

Q^P : Set of cost for CS^P

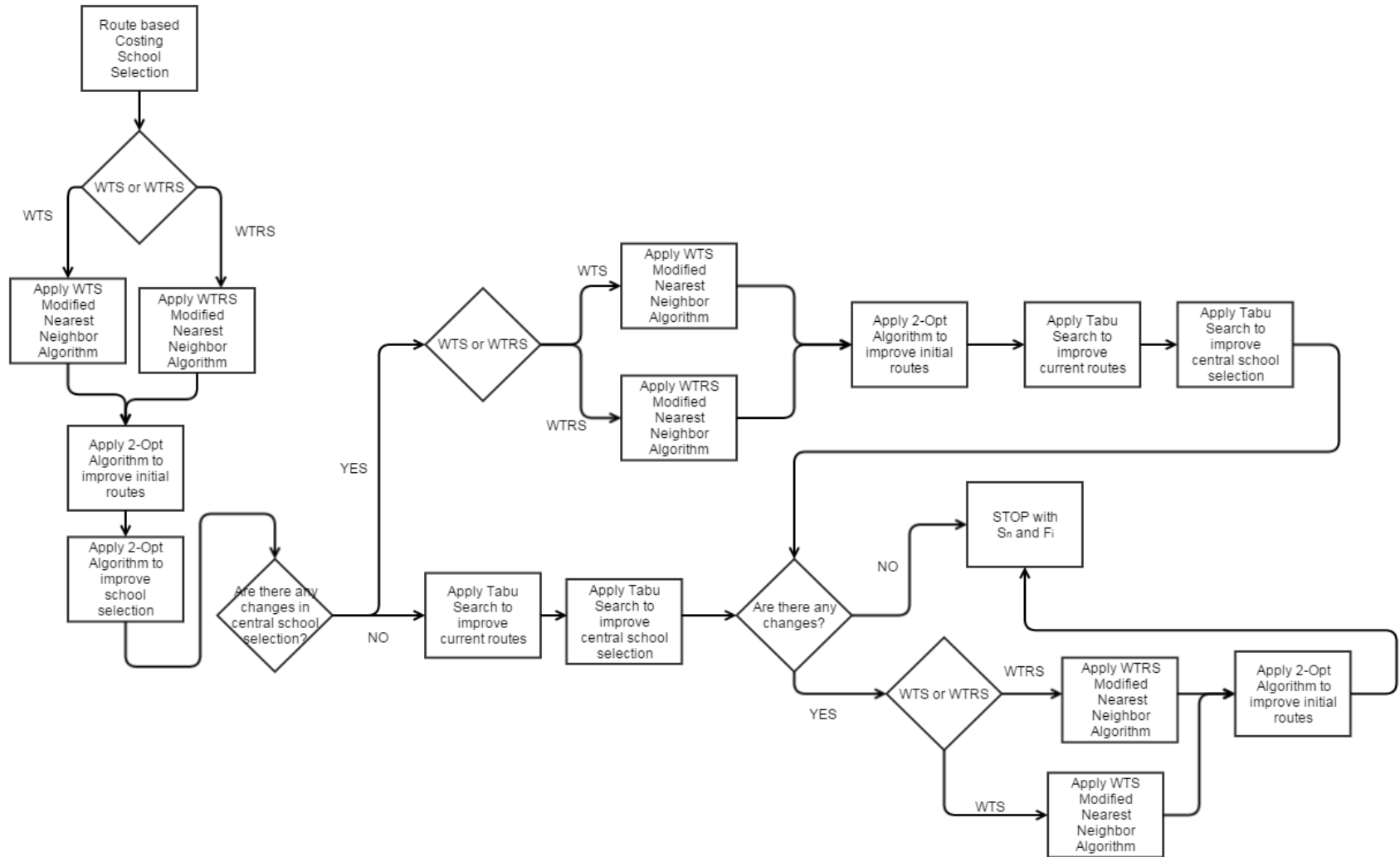
CS_r^P represents the r^{th} P-tuple of CS^P

Q_r^P represents the r^{th} element of Q^P

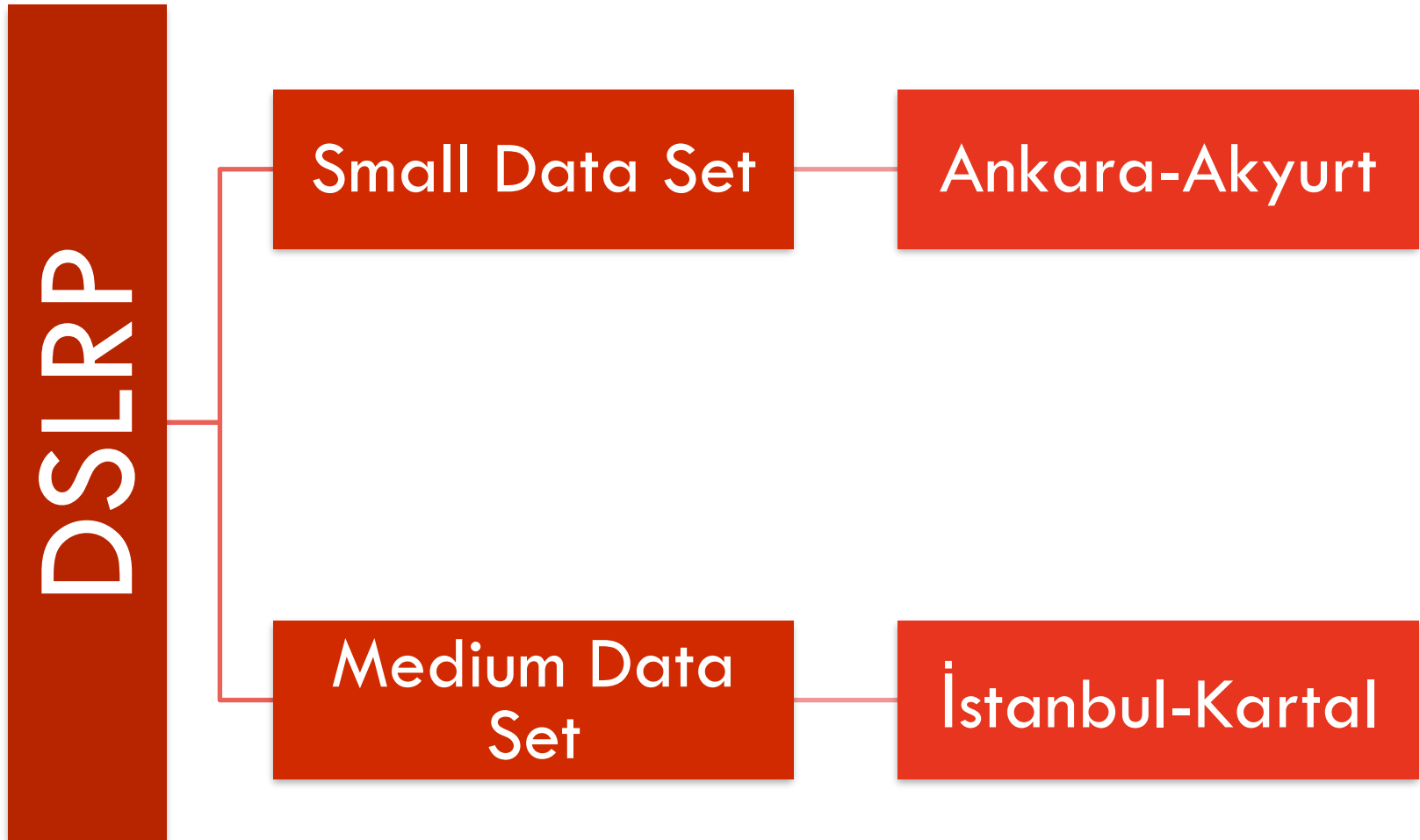
$|CS^P| = R$



Heuristics

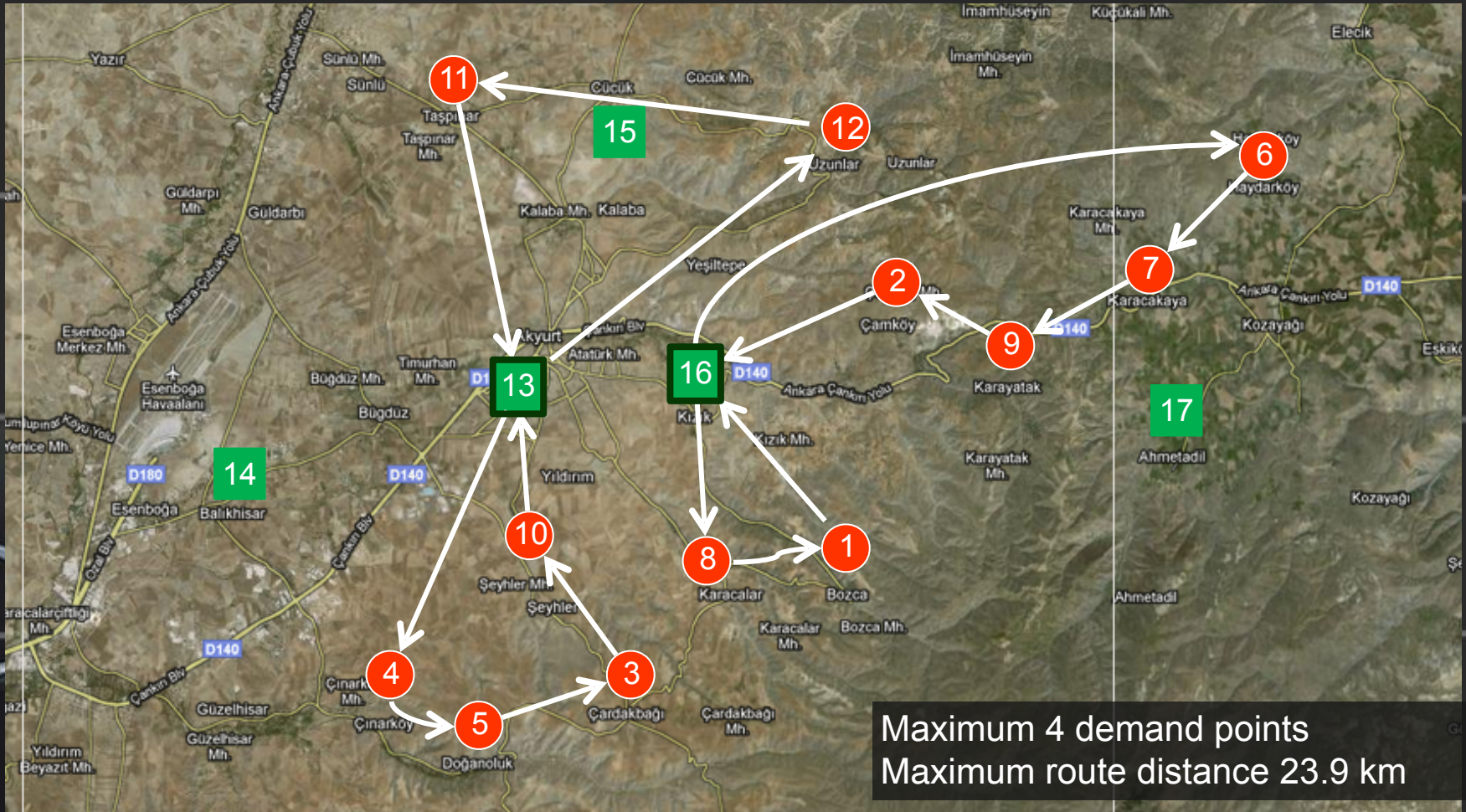


Data Sets & Computational Analysis



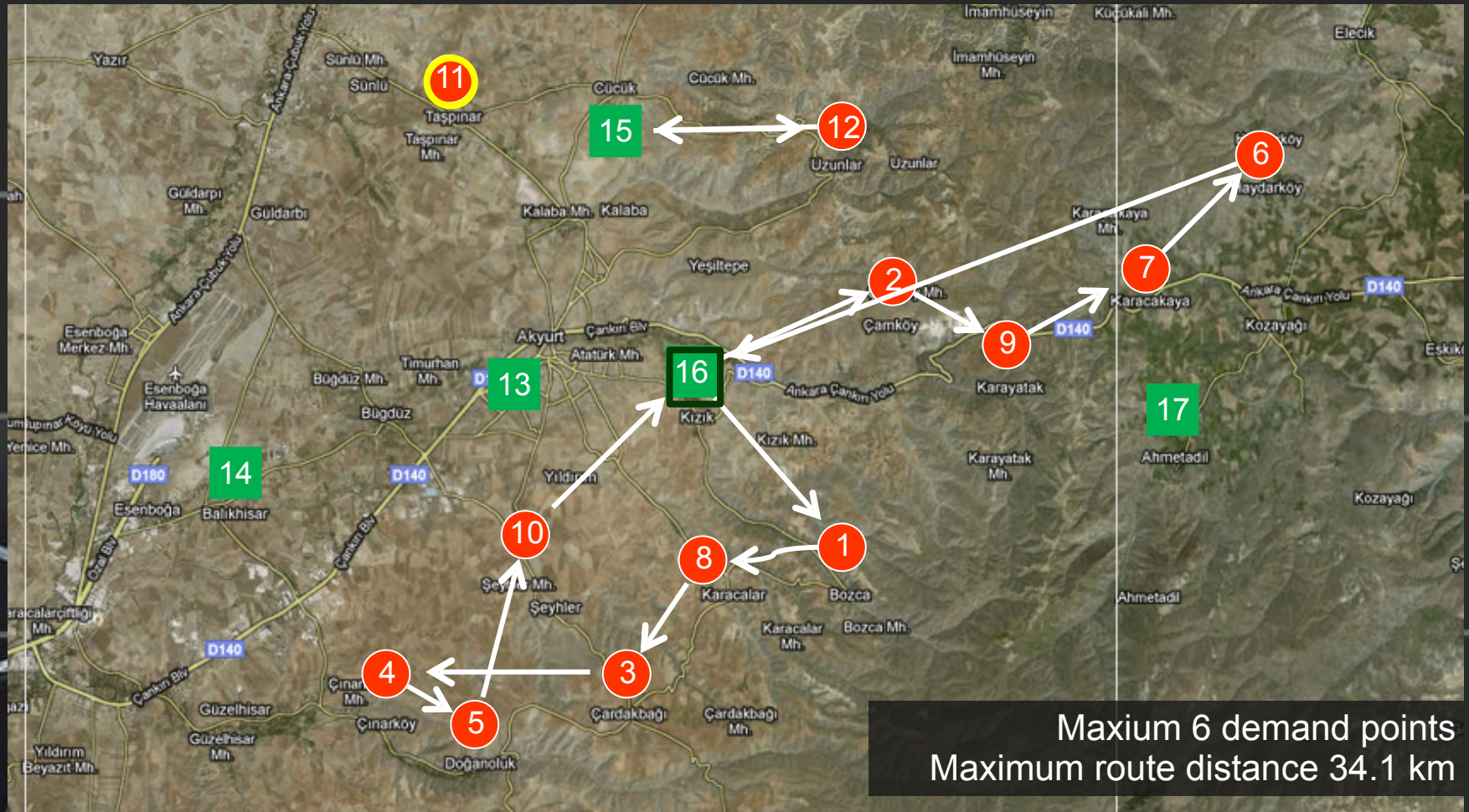
Results

D- WTS
 $\alpha=4$, $\gamma=2$, $p=2$, $B=2$



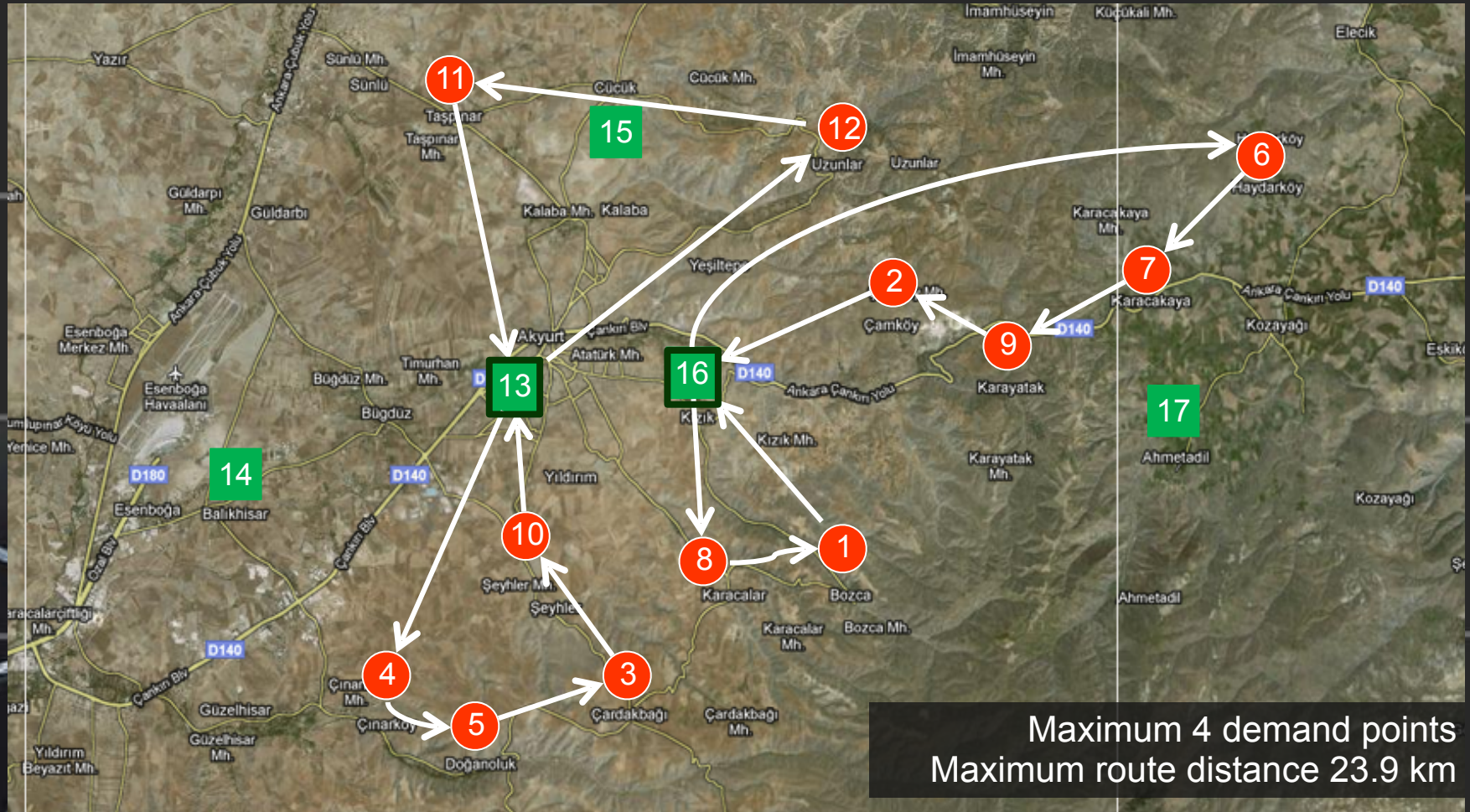
Results

Cum WTRS
 $\alpha=4$, $\gamma=2$, $R=50$



Results

D-Cum WTRS
 $\alpha=4$, $\gamma=2$, $B=2$, $R =50$



Computational Results | Ankara -Akyurt

Instance #	Parameters			Result			
	α	γ	p	z	V	Central School Node Number	S
Instance 16	2	2	1	3312468	1 route	14	-
Instance 17	4	2	1	3260079	1 route	15	11
Instance 18	4	4	1	2836818	1 route	14	1,2,5,10
Instance 19	6	2	1	3260079	1 route	15	11
Instance 20	6	4	1	2836818	1 route	14	1,2,5,10
Instance 21	6	6	1	2607417	1 route	15	1,2,5,6,8,11
Instance 22	2	2	2	3001599	3 routes	14,15	-
Instance 23	4	2	2	2775429	2 routes	14,15	11
Instance 24	4	4	2	2458791	2 routes	14,15	2,5,10,11
Instance 25	6	2	2	2775429	2 routes	14,15	11
Instance 26	6	4	2	2458791	2 routes	14,15	2,5,10,11
Instance 27	6	6	2	2378016	2 routes	15,16	1,2,5,6,8,11
Instance 28	2	2	3	3001599	3 routes	14,15,16	-
Instance 29	4	2	3	2775429	2 routes	14,15,16	11
Instance 30	4	4	3	2458791	2 routes	14,15,17	2,5,10,11
Instance 31	6	2	3	2775429	2 routes	13,14,15	11
Instance 32	6	4	3	2420019	2 routes	14,15,16	1,2,5,8,10,11
Instance 33	6	6	3	2210004	3 routes	14,15,16	1,2,5,6,8,11

Computational Results I Ankara – Akyurt

WTS

P	α	Assignment based Costing Heuristics %	Route based Costing Heuristics %
1	1	10,86965161	2,431906615
1	2	10,86965161	2,431906615
1	4	12,95738539	4,360753221
2	1	22,2681644	10,61104431
2	2	22,2681644	10,61104431
2	4	32,23180993	10,66036998

AVERAGE		18,57747122	6,851170843
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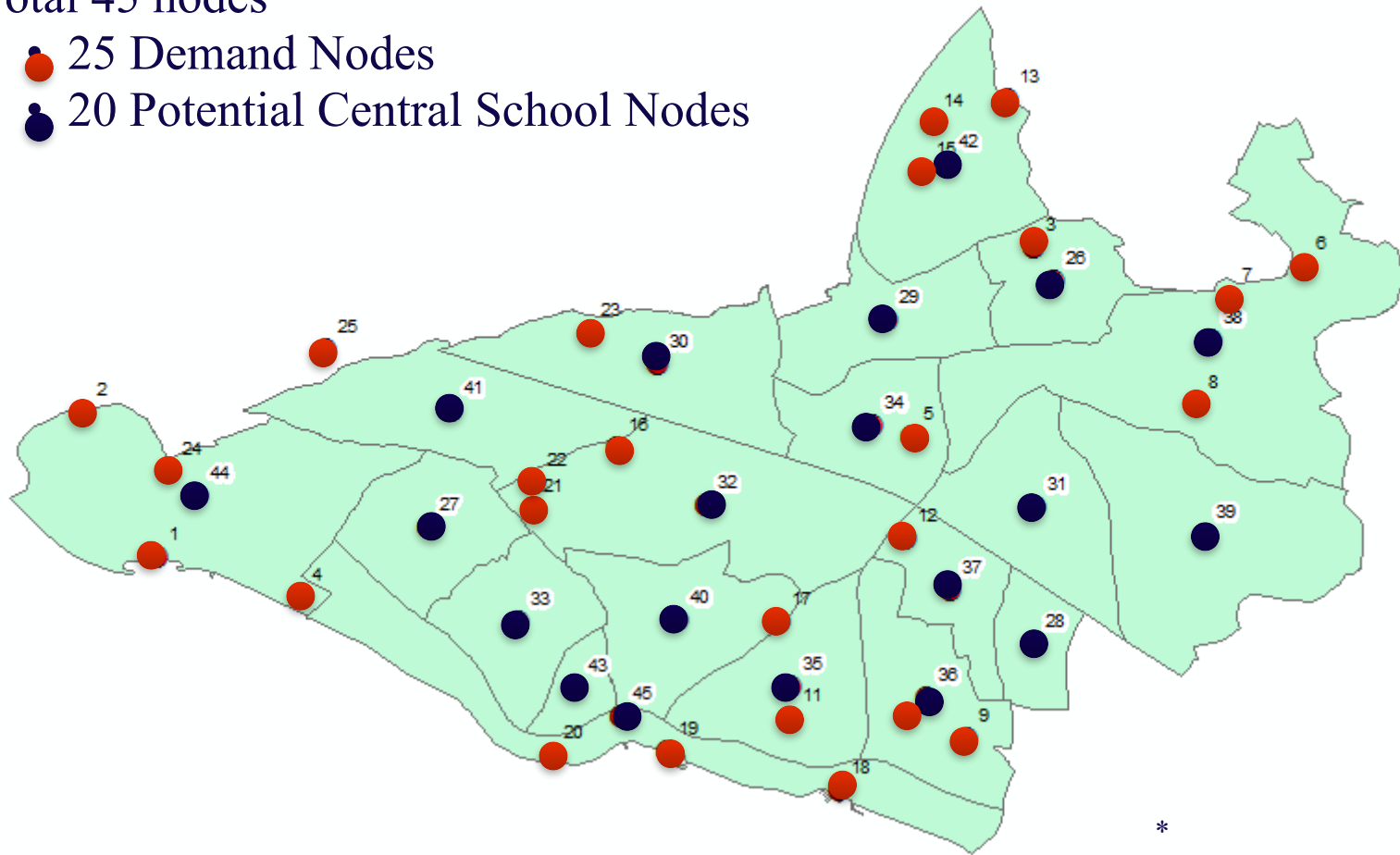
WTRS

P	α	γ	Assignment based Costing Heuristics %	Route based Costing Heuristics %
1	2	2	16,52942761	2,7102149
2	2	2	22,2681644	10,611044
3	2	2	17,9371395	8,9547938
1	4	2	18,40203872	4,3607532
2	4	2	32,23180993	10,66037
3	4	2	27,54784936	8,1490105
1	4	4	31,13283968	9,7949886
2	4	4	48,44693998	12,746468
3	4	4	28,51844667	8,0157687
1	6	2	18,40203872	7,333902
2	6	2	32,23180993	6,6357669
3	6	2	27,54784936	10,82647
1	6	4	31,13283968	8,5420355
2	6	4	48,44693998	13,271929
3	6	4	30,5774872	9,7462458
1	6	6	42,66992967	4,9567445
2	6	6	35,82751336	2,9892145
3	6	6	23,98167605	11.988485

AVERAGE			29,65737443	8,4607892
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Data I İstanbul- Kartal

- Total 45 nodes
 - 25 Demand Nodes
 - 20 Potential Central School Nodes



Computational Results I İstanbul – Kartal

			Assignment Based Costing			Route Based Costing		
P	α	γ	Objective	Central School Node Number	S	Objective	Central School Node Number	S
2	2	2	14,1	40,26	1,4,5,7,9,10,12,13,14,15,16,18,20,21,22,24	10,1	42,26	1,4,5,6,7,8,9,10,12,13,14,15,16,18,20,21,22,23,24
3	2	2	13,81	40,26,44	1,2,4,5,7,8,9,10,12,14,15,16,21,22,23,25	8,88	26,42,30	1,2,4,5,6,7,8,8,10,12,13,14,15,16,18,20,21,22,23,25
2	2	-	18,73	40,26	1,4,5,6,8,9,10,14,15,16,18,20,21,22,24,23	13,97	30,26	1,4,5,6,8,9,10,12,13,14,15,16,18,20,21,22,24,23
3	2	-	15,17	40,26,44	1,2,4,5,6,12,13,14,15,16,18,20,21,22,23,25	11,21	32,26,42	1,2,4,5,6,7,9,10,12,13,14,15,16,18,20,21,22,23,25

Conclusion

School Districting Application

Demand-selective LRP

Cumulative and Distance-constrained versions

Two real life scenarios, Akyurt-Ankara and Kartal-Istanbul

Mathematical Models and Two different heuristics approaches





Thank You for Listening Questions & Answers

