1. (30 points) The inventory of a purchased item is controlled with a continuous review (Q, R) policy and a lead-time of two weeks. The pdf $f(x)$, the cdf $F(x)$, and the loss function $n(y) = E[\max(x - y, 0)]$ of the lead-time demand are as follows:

$$f(x) = \begin{cases} 1/3200 & \text{if } 0 \le x \le 800, \\ 1/1600 & \text{if } 800 \le x \le 2000. \end{cases}$$

$$F(x) = \begin{cases} x/3200 & \text{if } 0 \le x \le 800, \\ (x - 400)/1600 & \text{if } 800 \le x \le 2000. \end{cases}$$

$$n(y) = \begin{cases} 1150 - y + \frac{y^2}{6400} & \text{if } 0 \le y \le 800, \\ 1250 - \frac{5y}{4} + \frac{y^2}{3200} & \text{if } 800 \le y \le 2000. \end{cases}$$

The expected lead-time demand is 1150 units. The fixed ordering cost is \$300 and the unit holding cost per week is \$0.16. Shortages are backordered at a cost of \$5 per unit. Assume 50 weeks in a year. All numbers in your solutions should be rounded with no digit after the decimal point.

(a) (7 points) Find the optimal reorder level if the optimal order quantity is 1538 units.

ANSWER: Recall the optimality condition:

$$1 - F(R) = \frac{Qh}{p\lambda} = \frac{(1538)(0.16)(50)}{(5)(1150)(25)} = 0.0856.$$

This implies that $F(R) = 0.9144$ and $R = (1600)(0.9144) + 400 = 1863$.

(b) (8 points) Find the proportion of demands that are met from stock (the fill rate) for the solution in part (a).

ANSWER: Calculate β:

$$\beta = 1 - \frac{n(R)}{Q} = 1 - \frac{n(1863)}{1538} = 1 - \frac{5.86}{1538} = 0.996.$$

(c) (10 points) Find the optimal reorder point if the lead-time demand is incorrectly assumed to be a constant at 1150 units. Use the deterministic EOQ problem with no backorders. Find the fill rate for this solution if the correct lead-time demand distribution is used.

ANSWER: We first calculate the EOQ value for the deterministic problem:

$$Q = \sqrt{\frac{2K\lambda}{h}} = \sqrt{\frac{2(300)(1150)(25)}{(0.16)(50)}} = 1468.$$

Since $1150 < 1468$, $R = 1150$. Thus:

$$\beta = 1 - \frac{n(R)}{Q} = 1 - \frac{n(1150)}{1468} = 1 - \frac{226}{1468} = 0.846.$$

(d) (5 points) Find the reorder level if the probability of no stock-out in an ordering cycle is stated as 0.90.

ANSWER: Since $F(R) = 0.90$, $R = (1600)(0.90) + 400 = 1840$.

2. (25 points) The Ministry of Health has established the following procedure for COVID-19 testing: Upon arrival at a hospital, each patient goes through three steps that should be administered by a nurse: (A) taking a nasal swab sample for PCR testing, (B) taking a blood sample for antibody testing, and (C) taking a chest computed tomography scan. These steps should be in the order A – B – C. Consider a hospital that has two nurses (X and Y) and four patients (1, 2, 3, and 4) waiting for COVID-19 testing. The durations of three steps for these patients are:

Patients	A	B	C
1	5	3	6
2	7	2	5
3	4	1	4
4	6	4	7

(a) (7 points) If X is responsible from A and B, and Y is responsible from C, find the sequence of patients that minimizes the makespan and compute the resulting makespan. Each patient, upon completion of A, should immediately proceed with B.

ANSWER: If we apply Johnson's algorithm to the following table, we find the optimal sequence: 4 – 1 – 2 – 3. The makespan for this sequence is 36.

Patients	A + B	C
1	8	6
2	9	5
3	5	4
4	10	7

(b) (7 points) If X is responsible from A, and Y is responsible from B and C, find the sequence of patients that minimizes the makespan and compute the resulting makespan. Each patient, upon completion of B, should immediately proceed with C.

ANSWER: If we apply Johnson's algorithm to the following table, we find the optimal sequence: 3 – 1 – 4 – 2. The makespan for this sequence is 36.

Patients	A	B + C
1	5	9
2	7	7
3	4	5
4	6	11

(c) (5 points) Find the mean flow time for the sequence 1 – 2 – 3 – 4 if X is responsible from A and B, and Y is responsible from C.

ANSWER: The flow times are 14, 22, 26, and 39. Thus the mean flow time is 25.25.

(d) (6 points) The makespan is 39 for the sequence in part (c). The hospital management now considers assigning X to A and C, and Y to B, and re-optimizing the sequence. Does this new assignment help obtain a makespan value less than 39? Why or why not?

ANSWER: No. Since the total nursing time of X is 44 in the new assignment, the makespan cannot be lower than 44.

Question 1 (30 points)

a) Calculation of F(R) or n(R) from the first order conditions: 5 points (formula has one point and each input value has one point).
Calculation of R: 2 points.

b) Formula of beta: 2 points.
Input value of Q: 2 points.
Input value of R: 2 points (if R is wrong in part a, no further points deducted).
Calculation of n(R): 2 points.

c) Calculation of EOQ: 4 points (formula has one point and each input value has one point).
Stating R = 1150: 2 points.
Calculation of beta: 4 points (formula has one point, input values of Q and R have two points, and calculation of n(R) has one point).

d) Stating F(R) = 0.90: 2 points.
Calculation of R from the distribution: 3 points.

Question 2 (25 points)

a) Summation of A + B: 2 points.
Optimal sequence: 3 points (mention of Johnson but wrong sequence: -2, correct sequence but no table or indication of Johnson: -3).
Makespan: 2 points (optimal sequence is wrong but makespan calculation is correct: full credit).

b) Summation of B + C: 2 points.
Optimal sequence: 3 points (mention of Johnson but wrong sequence: -2, correct sequence but no table or indication of Johnson: -3).
Makespan: 2 points (optimal sequence is wrong but makespan calculation is correct: full credit).

c) Flow times of the four jobs: 4 points.
Calculation of the mean flow time: 1 point.

d) The answer "No": 1 point.
Correct explanation: 5 points (incomplete explanations such as mentioning only that A and C are longest operations or providing a schedule with makespan 44 without stating it is optimal: -3).

IE 375 Final Exam Section 01 January 2, 2021 Part 2 - Solutions

Question 3 (20 points)

Define investment in inventory, **II**, as the average inventory carried multiplied by unit cost.
a) (7 points) What is the total present investment in inventory (sum of all items). Note that you need to find the average inventory carried for each item given that 4 orders are placed for each item every year.

Average inventory carried is Q/2. Q for each item is a quarter of annual demand. Hence, the following table can be prepared:

Item	Q (units)	Cost ($/unit)	Investment ($)
1	2 500/4	4	1 250
2	3 200/4	2	800
3	500/4	5	312.5

Total Investment= 1250+800+312.5=$2362.5

Grading: Correct average inventory: +4, correct investment +3

b) (13 points) Keeping the average investment level as found in **a)**, write a model to find the minimum total number of orders that can be placed to determine the order size for each item under the new policy. (Hint: You must formulate a problem where the objective function is to minimize total number of orders, subject to a constraint on the average value of the inventory.) DO NOT solve the problem.

Min $2500/Q_1 + 3200/Q_2 + 500/Q_3$
s. to: $c_1Q_1/2 + c_2Q_2/2 + c_3Q_3/2 \leq 2362.5$ where c_i's are unit costs; 4, 2, 5 for i=1, 2, 3 respectively.
$Q_i > 0$ for all i

Grading: Correct objective: +7, correct investment +3, correct RHS +2, non-negativity +1

Question 4 (25 points)

IE has come up with the following constraints: $W_t = W_{t-1} + H_t - F_t$, $W_0 = w$, for $1 \le t \le 6$.

Here, W_t is the workforce level in the beginning of the month t, H_t and F_t are the number of workers hired and fired in the beginning of the month t.

a. **(8 points)** Define explicitly all the new variables you may need to use.

These are decision variables: S_{At}: Number of units bought from subcontractor A in period t, t=1, 2, …, 6
S_{Bt}: Number of units bought from subcontractor B in period t, t=1, 2, …, 6
P_t: Number of units produced in house in period t, t=1, 2, …, 6
I_t: Number of units in the inventory at the end of period t, t=1, 2, …, 6; I_0=0, I_6=0

These are parameters: $R: Wage for a worker/month, L: Number of units which can be subcontracted from A.

Grading: Each decision variable +2 points. -1 for missing t values – at most -2 for the part.
If decision variable definitions are contradicting: You may have partial grades which would be less than what the above line states.
For each binary variable defined: -2
No grades for parameters

b. **(5 points)** Write the objective function of the model.

$\sum_{t=1,..,6}(c_H H_t + c_F F_t + c_R P_t + c_A S_{At} + c_B S_{Bt} + c_I I_t + R W_t)$

Grading: Each +1 up-to 5 of them. If all six are available (with definition R) +2 Bonus

c. **(12 points)** What other constraints should be in the model?

Inventory Balance: $I_t = I_{t-1} + P_t + S_{At} + S_{Bt} - D_t$ t=1, 2, …, 6
Manpower/production relation: $P_t \le K W_t$ t=1, 2, …, 6
Capacity Limitation for subcontractor A: $S_{At} \le L$ t=1, 2, …, 6
Non-negativity for all decision variables *W, H, F, S, P and I*

Grading: Inv. Balance +6 (each term missing -1 up-to two terms, if more is m; issing depends); . -1 for missing t values.
Manpower/Production: +3 (If K is missing at most +1); . -1 for missing t values
Subcontractor constraint: +2
Non-negativity: +1
Any redundant constraint may be penalized depending on the implication for general understanding.